PROPERTIES OF THIN LENSES

**Object:** To measure the focal length of lenses, to verify the thin lens equation and to observe the more common aberrations associated with lenses.

**Apparatus:** PASCO Basic Optical Bench, Light Source, Light Stand, and Power Supply, Optics Viewing Screen, 100 and 200 mm Lens mounted in protective Lens Holders, Acrylic Concave and Convex Lens, Acrylic Rhombus, Black Ray Blocking Cylinder, 30 centimeter ruler, white paper.

**Theory:**

**Paraxial-ray Equations**

When light rays traverse a thin lens at small angles to the lens axis, the object and image distances are given by the thin-lens equation:

\[
\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}
\]  \hspace{1cm} (1)

The magnification is defined as the ratio of image height \((y')\) to object heights \((y)\)

\[
m = \frac{y'}{y}
\]  \hspace{1cm} (2)

Where heights above the optical axis are taken as positive and heights below the axis are negative.
It can be shown that the magnification can also be expressed as:

\[ m = -\frac{d_i}{d_o} \]  

(3)

where \( d_o \) and \( d_i \) are the object and image distances for a lens of focal length \( f \). The focal length, \( f \), is determined by the radii of curvature, \( R_1, R_2 \), of the lens surfaces, and by the refractive index, \( n \), of the glass. For a lens with index of refraction \( n \), in air \((n=1)\) the lens makers' formula looks like:

\[ \frac{1}{f} = (n - 1) \cdot \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \]  

(4)

If there is some confusion concerning ray and lens conventions, please consult your instructor.

**Aberrations**

If the light rays make appreciable angles with the lens axis, or are a large distance from the axis of the lens, deviations from formulas (1) and (3) occur. These geometrical aberrations are not due to flaws in the lens, but are a consequence of Snell’s law of refraction, and the fact that thin lenses are almost always ground with spherical surfaces.

Furthermore the refractive index, used in Eq. 4, varies with wavelength and therefore the focal length will be appreciably different for different colors in the spectrum. This chromatic aberration gives rise to the colored edges of images formed by simple lenses.

**Equipment Description:**

Mounting the Light Source to the Optical Bench: The Light Source is held in place by the spring action of the bracket. See Figure 1. The power cord from the transformer must be unplugged from the Light Source before taking the bracket off or on. To attach the bracket, hold the Light Source box in one hand and pull outward on each of the bracket's two sides and insert the light box's side tabs into the holes in the sides of the bracket. Attach the bracket to the optics bench by loosening the bracket thumbscrew and inserting the nut into the T-slot in the center of the track. Rotate the Light Source box in its bracket until its label side is up and the box clicks into place. In this horizontal position, the Light Source acts as a cross-arrow object.
**Using the Light Source as a Ray Box:** To use the Light Source as a Ray Box, remove the bracket. Set the Light Source on a piece of white paper. Setting the Light Source on the table with the label side up produces white light rays. Slide the plastic mask to select the number (1, 3, 5) of white rays. If the label side is down, it will produce the three primary colors.

**Mounting the Screen and Lenses to the Optics Bench:** To mount the viewing screen to the Optical Bench, loosen the thumbscrew and slide the square nut into the center T-slot in the bench. See Figure 2. The lenses simply snap into place on the bench. See Figure 3. To slide the lens holder along the bench, grasp it at its base and squeeze the locking clip on the side of the holder.

**Procedure:** Please handle lenses, diaphragms and filters by the edges; avoid leaving fingerprints!

**A. Focal Length:**

1. Place the Ray Box on a white piece of paper. Using five white rays from the Ray Box, shine the rays straight into the convex lens. See Figure 4. Trace around the surface of the lens and trace the incident and transmitted rays. Indicate the incoming and the outgoing rays with arrows in the appropriate directions.

2. The place where the five refracted rays cross each other is the focal point of the lens. Measure the focal length from the center of the convex lens to the focal point.

3. Repeat the procedure for the concave lens. Note that in Step 2, the rays leaving the lens are diverging and they will not cross. Use a ruler to extend the outgoing rays straight back through the lens. The focal point is where these extended rays cross.

4. Nest the convex and concave lenses together and place them in the path of the parallel rays. Trace the rays. What does this tell you about the relationship between the focal lengths of these two lenses?

5. Slide the convex and concave lenses apart to observe the effect of a combination of two lenses. Then reverse the order of the lenses. Trace at least one pattern of this type.

6. Place the convex lens in the path of the five rays. Block out the center 3 rays (using the black cylindrical shaped block) and mark the focal point for the outer two rays. Next, slide the mask to the position that gives only the 3 inner rays and mark the focal point. Are the two focal points the same? The difference between these focal lengths is called the **longitudinal spherical aberration**: express it as a percent of the mean image distance. Note that the best focus is in fact a compromise position.
B. Determine the Refractive Index Using Apparent Depth

See Figure 5a. Light rays originating from the bottom surface of a block of transparent material refract at the top surface as they emerge from the material. When viewed from above, the apparent depth, d, of the bottom surface of the transparent material is less than the actual thickness, t. The apparent depth is given by d = t/n, where n is the index of refraction of the material.

Measure the apparent depth of the rhombus by using the Light Source as a Ray Box. Using five white rays, shine them straight into the convex lens. See Figure 5b. Use the black cylinder to block the center 3 rays. Mark the place where the outer two rays cross. Place the rhombus as shown in Figure 5b. The bottom surface must be exactly where the two rays cross. The crossed rays simulate the rays that emerge from the bottom of the rhombus (Figure 5a). Trace the top and bottom surface of the rhombus. Trace the inner and outer edges of the rays that emerge from the top of the rhombus. Remove the rhombus and trace the diverging rays back to their crossing points. Measure the apparent depth of the inner and outer edges of these rays and average their distances. Calculate the index of refraction using n=t/d. Calculate the % error between your value and the index of refraction for Acrylic (n=1.497).

C. Chromatic Aberration

Setup the Light Source and adjust the Ray Box so the primary colors are showing. When light crosses the surface of the lens or rhombus it is refracted. Because the index of refraction varies with frequency, different colors of light will be refracted at different angles. The triangular end of the rhombus is used as a prism in this experiment. See Figure 6. Keep the colored rays near the point of the rhombus for maximum transmission of light. Rotate the rhombus until the angle of the emerging rays is as large as possible. Do the colored rays emerge from the rhombus parallel to each other? Why or why not? How would this effect the focal point of a lens?

Place the convex lens near the Ray Box so it focuses the rays and causes them to cross each other at the focal point. What is the color of the light where the three rays come together?
D. Lensmaker's Equation

In this section you will determine the focal length of the concave lens using equation (4). To do this you will measure the two radii of curvature $R_1$ and $R_2$. Set up the Light Source as a Ray Box, put the concave lens in the path of the rays and observe the faint reflected rays off the first surface of the lens. The front of the lens can be treated as a concave mirror having a radius of curvature equal to twice the focal length of the effective mirror. Trace the surface of the lens and the incident rays and the faint reflected rays. Measure the distance from the center of the front curved surface to the point where the faint reflected rays cross. The radius of curvature of the surface is twice this distance. Note that the lens is symmetrical and it is not necessary to measure the curvature of both sides of the lens because $R$ is the same for both. Calculate the focal length of the lens using the equation (4). The index of refraction is 1.497 for the Acrylic lens. Remember that a concave surface has a negative radius of curvature.

E. Focal Length of a Thin Lens

Using either the 100 or 200 mm lens, the Light Source, and the Screen, set up the Optical Bench as shown if figure 7. Be sure the object (the cross-arrow on the front of the Light Source) and the screen are at least one meter apart. Move the lens to a position where an image of the object is formed on the screen. Measure the image distance and the object distance. Measure the object size and the image size for this position of the lens. Move the lens to a second position where the image is in focus (do not move the screen or Light Source). Measure the image distance and the object distance. Measure the image size for this position also. Calculate the focal length of the lens using equation (1) for both lens positions. Take the average of these two focal lengths. Calculate your percent error.

Figure 7

For both sets of data points, use image and object distances (Figure 7) to find the magnification at each position of the lens using equation (3). Then using your measurements of the image size, find the magnification using equation (2). Find the percent differences.

Questions

1) Is the image formed by a lens upright or inverted?
2) In the Image real or virtual? How do you know?
3) Explain why for a fixed object-to-screen distance, there are two lens positions where a clear image is formed.
4) By looking at an image, how can you tell that the magnifications is negative?