CONSERVATION OF MOMENTUM

Object: To study the law of conservation of momentum by analyzing collisions in two dimensions.

Apparatus: Digital scale, Ramp, two steel balls, one hollow or plastic ball, C-clamp, plumb bob, sheet of easel paper (50cm x 50cm), masking tape, meter stick, protractor, carbon-paper (2 sheets).

Theory: The momentum of an object is defined as \( p = m \mathbf{v} \) where \( m \) is the mass and \( \mathbf{v} \) the velocity. If ball A with momentum \( p \) (figure #1) strikes ball B at rest, and if the momenta of balls A and B are \( p_A \) and \( p_B \) after the collision, then conservation of momentum requires that

\[
\mathbf{p} = \mathbf{p}_A + \mathbf{p}_B \quad (1)
\]

Note: This is a vector relation so directions are involved. Since momentum is conserved this implies that the vector quantities (i.e. \( \Sigma p_x \)'s and \( \Sigma p_y \)'s) must be conserved. A typical collision is sketched in the figure below.

![Figure #1](image1.png)

The vector diagram for the collision looks like:

![Figure #2](image2.png)

In this experiment equation #1 will be verified for several different kinds of collisions. In order to do so, \( \mathbf{p}, \mathbf{p}_A \), and \( \mathbf{p}_B \) must be measured. Consider first a collision in which balls A and B have the same mass. Since both A and B have the same mass, and B is initially at rest it can be shown that after the collision A and B will leave at right angles to each other, \( 90^\circ \). The mass then cancels out of equation #1 leaving

\[
\mathbf{v} = \mathbf{v}_A + \mathbf{v}_B \quad (2)
\]
so we need not be concerned with the value of the mass.

**Conservation of Momentum**

The velocities \( v, v_A, \) and \( v_B \) are found by using the apparatus provided in the following way.

Let the ball roll down the ramp from the top and strike the floor. Once the ball clears the ramp, the horizontal component of its velocity does not change until it strikes the floor, so that the distance \( s \) in the figure is given by \( s = v t \). Now \( t \), the time in flight, is not known, but it is not necessary to know it because it is constant for all balls taking part in a collision, and so by measuring \( s \), one has a measure of \( v \). The same technique is used to get a measure of \( v_A \) and \( v_B \). After a collision the displacements of the two balls are measured. To verify Eq. #2 then, it is written as

\[
vt = v_A t + v_B t
\]

or

\[
s = s_A + s_B
\]

Where \( s, s_A, \) and \( s_B \) are measured directly. When collisions between the steel ball and the hollow or plastic ball are considered, the masses must be retained in Eq. #1

**Procedure:**

A. Determination of \( s \). Secure the apparatus to the edge of the laboratory table with a C-clamp. Tape the large sheet of paper on the floor in front of the apparatus so the plumb bob point is near the center-edge of your paper. Align the plumb line so that it points straight down from the collision point. Mark this point on your paper. Roll a steel ball down the ramp and note where it strikes the paper. Now tape a piece of carbon paper, marking side down, so that the ball strikes about in the middle of it.

Now roll the ball down the ramp at least ten times, (10x), attempting to release it from the small height and in the same fashion each time. Remove the carbon paper and inspect your impact pattern, which should look something like this:
Use your best judgment to mark an X at the center of the pattern. Draw a line between the end of the plumb bob and the X. Measure and record the length of this line. This line represents \( s \). Circle the pattern and label it \( s \).

**B. Determination of \( s_A \) and \( s_B \).** Set the another steel ball in the first dimple on the collision plate. After noting where the balls hit the paper, place carbon paper at these impact positions as before and allow at least ten collisions (10 xs) to take place. Follow the procedure above in drawing lines to represent \( s_A \) and \( s_B \). Record the length of the lines and also the angles between \( s_A \) and \( s_B \) and between \( s_A \) and \( s \) and between \( s_B \) and \( s \). Label the impact patterns \( s_{1A} \) and \( s_{1B} \).

Repeat the experiment with the target placed in the second dimple, labeling the patterns \( s_{2A} \) and \( s_{2B} \).

Repeat for a collision in which a hollow or plastic ball is the target, (either dimple), labeling the patterns \( s_{3A} \) and \( s_{3B} \)

**Note: Treatment of Data (useful information)**

You need to determine the ratio mass of the hollow or plastic ball to that of the steel ball.

\[
\frac{m_{\text{hollow/plastic}}}{m_{\text{steel}}} = \text{ratio} = \#
\]

**Report:**

For each collision involving two steel balls, add the vectors \( s_A + s_B \) either graphically or analytically and compare the result to the initial vector \( s \). Make sure you include a rough diagram of the collision in your results if you choose the analytical method.

Do the same for the collision between the steel and hollow or plastic ball, only the measured vector \( s_B \) must be multiplied by the ratio (factor) given above. **(Why?)**

Give a percent error between the X-components for each trial and \( s \), the representative of the total momentum.
Give a percent error between the angles \([s_{1A}, s_{1B}), (s_{2A}, s_{2B})]\) of the first two trials (The Steel Balls), compared to the ideal angle of 90°.

\[S: \underline{\text{__________}}\]

<table>
<thead>
<tr>
<th>Collision</th>
<th>(S_A)</th>
<th>(S_B)</th>
<th>(\theta_A)</th>
<th>(\theta_B)</th>
<th>(S_{Ax} = S_A \cos \theta_A)</th>
<th>(S_{Ay} = S_A \sin \theta_A)</th>
<th>(S_{Bx} = S_B \cos \theta_B)</th>
<th>(S_{By} = S_B \sin \theta_B)</th>
<th>(%) difference between (S) and (S_{Ax} + S_{Bx})</th>
<th>(%) difference between 90° and (\theta_A + \theta_B)</th>
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Verify that:

a) \(S = S_A + S_B\)
b) \(S_x = S_{Ax} + S_{Bx}\)
c) \(S_y = S_{Ay} - S_{By}\)
QUESTIONS:

1) Do your experimental results support the conservation of linear momentum? Make an argument based on your results and % error.

2) What are some possible sources of error?

3) When is momentum conserved?

4) In what type of collisions is kinetic energy conserved?

5) Is kinetic energy conserved your experimental collisions, why or why not?

6) Why must the displacement vector of the lighter ball be scaled by the ratio of masses of the two balls?

7) Is it possible for either of the heavier balls to travel farther in the x direction than the control displacement S? What about the lighter ball? Why?