Critical Gap and Capacity of Roundabouts Based on Gap Acceptance Theory

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2 Calculation of Critical Gap
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1 Introduction of Roundabouts

- 1.1 History of roundabouts
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- 1.5 Some methods to calculate critical gap
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- 1.7 Calculation methods of roundabout capacity
1.1 History of roundabouts

In 1903, American researcher William P. Eno proposed the traffic circle, a traditional roundabout.

In 1905, the first circle in the world, the Columbus Circle in New York was built.

In the 1950s, increased traffic demand resulted in frequent deadlocks in bottlenecks where the entry vehicles have the priority and the circulating vehicles can't leave the intersection.

In 1966, yield rule of entry was applied. New kinds of roundabouts emerged in England, Australia and other countries.

In the early 1990s, American engineers named the intersection as modern roundabout which referred to the design idea of European countries and Australia.
1.2 Characteristics of modern roundabouts

- Entry's yield to circulation;
- Deviation of entry vehicles;
- Widen entry lanes.

Other characteristics:
- Splitter island; Good sight distance, traffic sign and natural light; No pedestrian crossing in circulatory roadway; No parking in roundabout area.
1.3 Gap acceptance theory

In general terms the gap acceptance process can be described as follows: drivers that want to make an operation, such as merging or overtaking, estimate the ‘space’ they need and estimate the available ‘space’. Based on the comparison between required and available space, they decide to start the manoeuvre or to postpone it. The term space can be expressed either in time or in distance.

- Unsignalized intersection controlled by priority;
- Vehicles on minor road use gaps between continuous vehicles on major road;
- Using queuing theory to describe the vehicle operation, calculate the capacity.

Major road: circulating flow; Minor road: entry flow
1.3 Assumption in gap acceptance model
(Catchpole, Plank, 1986; Hagring, 2000; Khattak, Jovanis et al; 1990)

(1) All gaps of circulating vehicle flows can be combined into gaps of single traffic flow;
(2) Gaps in major road obey a certain distribution;
(3) Gap acceptance is similar to lag acceptance;
(4) Entry vehicles arrive randomly;
(5) Absolute priority: vehicles in major road will not change operation when entry vehicles enter the intersection.
(6) When the circulating road vehicles exit the intersection before the point of conflict, the entry vehicle can identify them.
1.4 Definition of critical gap

- Critical gap $t_c$ can be defined as the threshold by which drivers in the minor stream judge whether to accept the gap or not. Drivers accept the gap $t$ when $t \geq t_c$, reject the gap $t$ and wait for the next gap when $t < t_c$.

- Including drivers' behavior, geometry of intersections, traffic flow, operation of vehicles and waiting time of drivers, there are a lot of factors which affect the acceptance or rejection maneuver (Hamet et al. 1997).

- Critical gap for different drivers follows a certain distribution, such as log-normal distribution (Troutbeck 1992) and hyper-Erlang distribution (Brilon 1995).

- However, for simplicity, many researchers assume that the critical gap is a fixed value (Troutbeck 1999, Tanyal et al. 2005).
1.5 Some methods to calculate critical gap

- Critical gap is defined as the gap for which the number of accepted shorter gaps is equal to the number of rejected longer gaps (Raff and Hart 1950, Drew 1968.)
- In 1999, some important methods for estimating critical gap were reviewed and a set of quality criteria was formulated (Brilon et al. 1999).
- Brilon et al. concluded that the maximum likelihood method (see Tian et al. 1999) and Hewitt’s method (Hewitt 1983) accurately calculate the critical gap.
1.6 Conceptions of capacity

- Highway capacity is defined by the Highway Capacity Manual as the maximum hourly rate at which persons or vehicles can be reasonably expected to traverse a point or a uniform segment of a lane or roadway during a given time period under prevailing roadway, traffic and control conditions.

- The capacity of an unsignalized control intersection always is defined as the capacity at the stop line. Capacity at entry or stop line is the emphasis because of the entry yield rule of modern roundabouts.
1.7 Calculation methods of roundabout capacity

- Weaving theory: the maximum weaving flow rate which traverses the weaving area, such as Wardrop equation.
- Gap acceptance theory: Entry capacity, major road intersect minor road in priority-controlled roundabout, using probability theory and queuing theory.
- Regression model: regression between circulating flow and entry capacity.
2 Calculation of Critical Gap

- 2.1 Assumption of gap acceptance in roundabout
- 2.2 When critical gap is a constant
- 2.3 When critical gap follows stochastic distribution
- 2.4 Two methods to calculate critical gap
- 2.5 Critical gap in emulation and reality
Definition of variables

- Suppose that the headway in the major stream follows M3 distribution.

\[
f(t) = \begin{cases} 
\alpha \lambda e^{-\lambda(t-t_m)}, & t > t_m \\
0, & t < t_m 
\end{cases} \quad (1)
\]

\[
F(t) = \begin{cases} 
1 - \alpha e^{-\lambda(t-t_m)}, & t \geq t_m \\
0, & t < t_m 
\end{cases} \quad (2)
\]

- $f(t)$ is the probability density function of headway between successive vehicles in major stream;
- $F(t)$ is the cumulative distribution function of headway;
- $\lambda$ is the decay constant (pcu/s); $t$ is the headway in major stream;
- $t_m$ is the minimum headway in major stream (s); $\alpha$ is the proportion of free vehicles.
2.1 Assumption of gap acceptance in roundabout

- It is independent between arrival times of the minor stream vehicles and the ones of the major stream vehicles

- The distribution form of all headway samples in major stream should be the same as the distribution form of part of samples when the vehicles in minor stream arrive before the intersection.

- So we can simulate the headway distribution in major stream by using the latter samples which were extracted when vehicles in the minor stream arrive before the intersection.

- These extracted headway samples can be divided into accepted headway and rejected headway because all surveyed headways are related to accepted maneuver or rejected maneuver.
2.2 When critical gap is a constant

- $\beta_a$ is the total accepted coefficient; the proportion of accepted gap number to total gap number.
- $\beta_r$ is the total rejected coefficient; the proportion of rejected gap number to total gap number. The relation between $\beta_a$ and $\beta_r$ is:
  \[ \beta_a + \beta_r = 1 \]  \hfill (3)
- So,

\[
\beta_a = P\{t \geq t_c\} = \int_{t_c}^{\infty} f(t) \, dt = 1 - F(t_c) = \alpha e^{-\lambda(t_c - t_m)} \]  \hfill (4)

\[
t_c = t_m - \frac{1}{\lambda} \ln\left(\frac{\beta_a}{\alpha}\right) \]  \hfill (5)
2.3 When critical gap follows stochastic distribution

- $f_{ao}(t)$ is the accepted proportion function, the possibility that minor stream vehicles accept the gap $t$ of major stream;
- $f_{ro}(t)$ is the rejected proportion function, the possibility that minor stream vehicles reject the gap $t$ of major stream.

$$f_{ao}(t) + f_{ro}(t) = 1 \quad (6)$$

$f_a(t)$ is probability density function of accepted gap;
$F_a(t)$ is cumulative distribution function of accepted gap;
$f_r(t)$ is probability density function of rejected gap; and
$F_r(t)$ is cumulative distribution function of rejected gap.
Derivation process

- Based on the measurement of gaps of major stream when vehicles in minor stream arrive before the intersection, samples including \( n_r \) rejected gaps and \( n_a \) accepted gaps can be obtained. Obviously,

\[
    n = n_r + n_a \quad (7)
\]

Assuming that gaps occur in the interval \((t, t+\Delta t)\) including \( m_r \) rejected gaps and \( m_a \) accepted gaps.

\[
    m = m_r + m_a \quad (8)
\]

So, total accepted coefficient \( \beta_a = \frac{n_a}{n} \), total rejected coefficient \( \beta_r = \frac{n_r}{n} \), and

\[
    \beta_a + \beta_r = 1 \quad (9)
\]
Derivation process

The following relations can be elicited when $\Delta t$ tends to be infinitely small.

$$f_{a0}(t) = \frac{m_a}{m}, \quad f_{r0}(t) = \frac{m_r}{m}$$

$$f_a(t)\Delta t = \frac{m_a}{n_a}, \quad f_r(t)\Delta t = \frac{m_r}{n_r}$$

$$f(t)\Delta t = \frac{m}{n}$$

$$f_a(t)\Delta t = \frac{m_a}{n_a} = \frac{m_a}{m} \cdot \frac{m}{n} \cdot \frac{n}{n_a} = \frac{f_{a0}(t)f(t)}{\beta_a} \Delta t$$

So the following equation can be deduced from equation (10).

$$\beta_a f_a(t) + \beta_r f_r(t) = f(t) \quad (11)$$

By taking integration on both sides of equation (11), it becomes

$$\beta_a F_a(t) + \beta_r F_r(t) = F(t) \quad (12)$$

$$f_{a}(t) = \frac{f_{a0}(t)f(t)}{\beta_a} \quad \text{and} \quad f_{r}(t) = \frac{f_{r0}(t)f(t)}{\beta_r} \quad (10)$$
Derivation process

The following equation can be elicited when \( t = t_c \) from equation (12).

\[
\beta_a F_a(t_c) + \beta_r F_r(t_c) = F(t_c) = \beta_r
\]

and

\[
F_a(t_c) = \frac{\beta_r}{\beta_a} [1 - F_r(t_c)] \quad (13)
\]

Raff (1950) considered that the number of rejected gaps larger than critical gap was equal to the number of accepted gaps smaller than critical gap.

Based on Raff’s definition of critical gap, the equation can be expressed as (Brilon, 1999)

\[
F_a(t_c) = 1 - F_r(t_c) \quad (14)
\]

In fact, equation (13) meets the meaning of Raff rather than equation (14).

The equation (13) and equation (5) are consistent with Raff’s definition.
2.4 Two methods to calculate critical gap

So, the critical gap can be described as follows:

(1) When the ratio of probability of accepted gaps not larger than a fixed value and probability of rejected gaps larger than such value is equal to the ratio of total rejected coefficient and total accepted coefficient, such a value is the critical gap; or

(2) Critical gap is the gap of major stream whose cumulative probability is equal to total rejected coefficient.

The two methods are in accordance with Raff’s definition of critical gap.
### 2.5 Critical gap in emulation and reality

Emulated values of rejected gaps and accepted gaps

<table>
<thead>
<tr>
<th>Order</th>
<th>$U_1$</th>
<th>Headway (initial)</th>
<th>Headway ($r=2, t &lt; t_w$)</th>
<th>Rejected proportion</th>
<th>$U_2$</th>
<th>State</th>
<th>Rejected gap</th>
<th>Accepted gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.61</td>
<td>1.96</td>
<td>2.00</td>
<td>1.00</td>
<td>0.64</td>
<td>0</td>
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<td></td>
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<tr>
<td>2</td>
<td>0.40</td>
<td>3.37</td>
<td>3.37</td>
<td>0.61</td>
<td>0.28</td>
<td>0</td>
<td>3.37</td>
<td></td>
</tr>
<tr>
<td>3</td>
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<td>0.68</td>
<td>2.00</td>
<td>1.00</td>
<td>0.37</td>
<td>0</td>
<td>2.00</td>
<td></td>
</tr>
<tr>
<td>4</td>
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<td>4.85</td>
<td>4.85</td>
<td>0.35</td>
<td>0.67</td>
<td>1</td>
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<tr>
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<td>2.62</td>
<td>2.62</td>
<td>0.80</td>
<td>0.29</td>
<td>0</td>
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</tr>
<tr>
<td>6</td>
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<td>3.70</td>
<td>3.70</td>
<td>0.54</td>
<td>0.50</td>
<td>0</td>
<td>3.70</td>
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</tr>
<tr>
<td>7</td>
<td>0.31</td>
<td>4.21</td>
<td>4.21</td>
<td>0.45</td>
<td>0.14</td>
<td>0</td>
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<td></td>
</tr>
<tr>
<td>8</td>
<td>0.75</td>
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<td>1.00</td>
<td>0.50</td>
<td>0</td>
<td>2.00</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.70</td>
<td>1.48</td>
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<td>0.63</td>
<td>0</td>
<td>2.00</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.91</td>
<td>0.62</td>
<td>2.00</td>
<td>1.00</td>
<td>0.09</td>
<td>0</td>
<td>2.00</td>
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<tr>
<td>11</td>
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<td>1.00</td>
<td>0.24</td>
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<td>2.00</td>
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</tr>
<tr>
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<td>0.38</td>
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<td>3.51</td>
<td>0.58</td>
<td>0.82</td>
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<tr>
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<td>0.36</td>
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<tr>
<td>15</td>
<td>0.05</td>
<td>10.16</td>
<td>10.16</td>
<td>0.05</td>
<td>0.53</td>
<td>1</td>
<td>10.16</td>
<td></td>
</tr>
</tbody>
</table>

U$_1$, U$_2$: random number in (0, 1);
State: rejected or accepted.
Accepted if U$_2$ is larger than rejected proportion and vice versa.

So, the rejected gaps and accepted gaps can be produced to calculate critical gap using different methods.
Critical gaps with different flow rates in emulation

$t_m = 2\, s, \, t_r = 3\, s, \, r = 0.365, \, secd = -1.$

<table>
<thead>
<tr>
<th>Order</th>
<th>$q$ (veh/s)</th>
<th>$\alpha$</th>
<th>M3 definition</th>
<th>Ashworth</th>
<th>Raff</th>
<th>Revised Raff</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.05</td>
<td>0.9</td>
<td>4.22</td>
<td>4.92</td>
<td>5.59</td>
<td>4.21</td>
</tr>
<tr>
<td>2</td>
<td>0.1</td>
<td>0.8</td>
<td>4.58</td>
<td>4.69</td>
<td>4.87</td>
<td>4.53</td>
</tr>
<tr>
<td>3</td>
<td>0.15</td>
<td>0.7</td>
<td>4.32</td>
<td>4.17</td>
<td>4.33</td>
<td>4.47</td>
</tr>
<tr>
<td>4</td>
<td>0.2</td>
<td>0.6</td>
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<td>3.81</td>
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<td>4.27</td>
</tr>
<tr>
<td>5</td>
<td>0.25</td>
<td>0.5</td>
<td>3.85</td>
<td>3.32</td>
<td>3.47</td>
<td>3.99</td>
</tr>
<tr>
<td>6</td>
<td>0.3</td>
<td>0.4</td>
<td>3.95</td>
<td>3.14</td>
<td>3.05</td>
<td>4.05</td>
</tr>
<tr>
<td>7</td>
<td>0.35</td>
<td>0.3</td>
<td>3.96</td>
<td>3.40</td>
<td>3.05</td>
<td>3.91</td>
</tr>
<tr>
<td>8</td>
<td>0.4</td>
<td>0.2</td>
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<td>4.01</td>
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<tr>
<td>9</td>
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<td>0.1</td>
<td>3.83</td>
<td>3.10</td>
<td>2.45</td>
<td>4.03</td>
</tr>
</tbody>
</table>

- Critical gaps using different methods decrease when flow rate increases, which is similar as reality.
- Drivers probably refuse larger gap when flow rate is low on major road because many large gaps can be used.
- However, drivers probably accept a smaller gap when flow rate is high on major road to avoid too long wait time, so the critical gap will be decreased.
- Results of M3 definition (Equ. (5)) and Revised Raff (Equ. (13)) are similar, they are close to Ashworth’s result.
Critical gap in the field

158 samples
(100 accepted gaps, 58 rejected gaps)

- Maximum Likelihood is an accurate method (Brilon, 1999). Its value was regarded as reference.
- Ashworth value is a little large, M3 definition is small, and the other two values are close to reference value.
- The revised Raff is the recommended method.
So many equations?

- Yes, we have deduced the new equations of critical gap.
- The equations of capacity can be deduced by using the new equation of critical gap based on gap acceptance theory.
- I don't want to introduce detail about the equations, just to list all kinds of capacity models and review them.
- Let's go!
3 Capacity Based on Gap Acceptance Theory

3.1 Basic equation of capacity in unsignalized intersection
3.2 Distribution of headway on major road
3.3 Capacity with different critical gap or function $g(t)$
3.4 Capacity of limited priority
3.5 Comparison of some methods
3.1 Basic equation of capacity in unsignalized intersection

Siegloch (1973)

\[ C = q \int_{t_0}^{\infty} f(t)g(t)\,dt \]

- For Continuous function \( g(t) \):
  \[ g(t) = \begin{cases} 
    \frac{t-t_0}{t_f}, & t > t_0 \\
    0, & t \leq t_0
  \end{cases}, \quad t_0 = t_c - \frac{t_f}{2} \]

- For discrete function \( g(t) \):
  \[ g(t) = nP_n(t), \quad n = 1, 2, \ldots, \infty \]

  \[ P_n(t) = \begin{cases} 
    1, & t_c + (n-1)t_f \leq t < t_c + nt_f \\
    0, & t < t_c
  \end{cases} \]
3.2 Distribution of headway in major road

- **M3**
  
  \[ f(t) = \begin{cases} 
  \alpha \lambda e^{-\lambda(t - t_m)} & t \geq t_m \\
  0 & t < t_m 
  \end{cases} \]

  \[ \hat{\lambda} = \frac{\alpha q}{1 - qt_m} \]

  \[ t_m = t_{(1)} = \min\{t_1, t_2, \ldots, t_n\} \]

  \[ t_m = 0 \quad \alpha = 1 \quad \lambda = q \]

- **M1** (negative exponential)

- **M2** (shifted negative exponential)

- **M3T**

  \[ t_m \neq 0 \quad \alpha = 1 - qt_m \quad \lambda = q \]

- The last three distributions are regarded as special M3 distributions

- Erlang distribution, Lognormal distribution, Mixed distribution

3.3 Capacity with different critical gap or g(t) function characteristics

3.3.1 Constant $t_c$, continuous $g(t)$

3.3.2 Constant $t_c$, discrete $g(t)$

3.3.3 $t_c$ follows a certain distribution
3.3.1 Constant $t_c$, Discrete function $g(t)$ (Troutbeck, 1986)

- M1 (Buckley et al, 1968)
  $$C = \begin{cases} \frac{q e^{-q t_c}}{1 - e^{-q t_f}}, & q > 0 \\ \frac{1}{t_f}, & q = 0 \end{cases}$$

- M2 (Luttinen, 2004)
  $$C = \begin{cases} \frac{e^{-\lambda(t_c-t_m)}}{1 - e^{-\lambda t_f}}, & q > 0 \\ \frac{1}{t_f}, & q = 0 \end{cases}$$

- M3T (Tanner, 1967)
  $$C = \begin{cases} \frac{(1-q t_m) q e^{-\lambda(t_c-t_m)}}{1 - e^{-\lambda t_f}}, & q > 0 \\ \frac{1}{t_f}, & q = 0 \end{cases}$$

- M3 (Guo, 2011)
  $$C = \frac{\beta a q}{1 - e^{-\lambda t_f}}$$
3.3.2 Constant $t_c$, Continuous function $g(t)$

- **M1** (Siegloch, 1973; refer to Wu, 1999)
  \[ C = \frac{e^{-qt_0}}{t_f} \]

- **M2** (Jacobs, 1980; refer to Wu, 2001)
  \[ C_p = \frac{e^{-\lambda(t_0-t_m)}}{t_f(1+\lambda t_m)}, t_0 > t_m \]

- **M3** (AUSTROADS, 1993; SIDRA, Akcelik, 1998)
  \[ C_p = \frac{\alpha q}{\lambda t_f} e^{-\lambda(t_0-t_m)}, t_0 > t_m \]
3.3.3 $t_c$ follows a certain distribution

Driver characteristics: consistent, homogeneous

(Readpole, Plank, 1986; Troutbeck, Brilon, 1997)

- The driver is consistent, the operation of vehicle is similar every time, so $t_c$ is constant.
- Drivers are homogeneous, their $t_c$ (Constant or Distribution) are similar. Otherwise, every $t_c$ is different.
- $t_c$ is regarded as stochastic distribution when nonhomogeneous or inconsistent.
3.3.3 $t_c$ follows a certain distribution

- Catchpole and Plank (1986)
- Heidemann and Wegmann (1997)
- Wu (2001)
- Guo (2011, 2014)
Catchpole and Plank (1986)

- Consistent, homogeneous: tc is constant, the equation is as former;
  \[ C = \frac{q(1 - qt_m)e^{qt_m}}{1 - \mathcal{L}(t_f(q))} \]

- Consistent, nonhomogeneous;

- Inconsistent, homogeneous: special situation as later equation;

- Inconsistent, nonhomogeneous.

\[ C = \frac{1}{\sum_j (\theta_j / q_j)} = \frac{\alpha qe^{\lambda t_m}}{1 - e^{-\lambda t_f}} \cdot \frac{1}{\sum_j \theta_j / \mathcal{L}(\theta_j(q))} \]

\( \theta_j \) is the proportion of the j kind of entry vehicle.

\( \mathcal{L}(\theta_j(q)) \) denotes the Laplace transformation of \( \theta_j \).
Heidemann and Wegmann equation

- Discrete function $g(t)$:

$$C = \frac{\lambda}{1 + \lambda B} \frac{L(t'_c(\lambda))}{1 - L(t_f(\lambda))} = \frac{\lambda}{1 + \lambda B} \frac{L(t_c(\lambda))L(t_m(-\lambda))}{1 - L(t_f(\lambda))}$$

- Continuous function $g(t)$:

$$C = \frac{\lambda}{1 + \lambda B} \frac{1}{1 - L(t_f(\lambda))L(t'_c(-\lambda))}$$

$$= \frac{\lambda}{1 + \lambda B} \frac{1}{(1 - L(t_f(\lambda))L(t_c(-\lambda))L(t_m(\lambda)))}$$

$B = \frac{t_m}{\alpha}$, $t'_c = t_c - t_m$;

$L(t'_c(\lambda))$ is the Laplace transformation of $t'_c$ at $\lambda$;

$L(t_c(\lambda))$ is the Laplace transformation of $t_c$ at $\lambda$;

$L(t_f(\lambda))$ is the Laplace transformation of $t_f$ at $\lambda$;

$L(t_m(\lambda))$ is the Laplace transformation of $t_m$ at $\lambda$. 
Circulating headway follow Erlang distribution:

\[ f(t_x) = \frac{\gamma}{(\alpha t_x - 1)!} (\gamma t_x)^{\alpha - 1} e^{-\gamma t_x} \]

(1) Discrete function \( g(t) \):
   
   Inconsistent operation:
   
   Consistent operation:
   
(2) Continuous function \( g(t) \)
   
   Inconsistent operation:
   
   Consistent operation:

\[ t_c, t_f, t_m \] follow shifted Erlang distribution.
Guo Equation

Assumption 1: circulating headway is M3 distribution, accepted gap interval is \((t_m, \infty)\)

\[
f_{r0}(t) = \begin{cases} 
e^{-r(t-t_m)}, & t \geq t_m \\ 0, & t < t_m \end{cases}
\]

Assumption 2: rejected proportion function is exponential function

- **Continuous** \(g(t)\):

\[
C = \alpha q \left[ \frac{r}{\lambda + r} + \frac{e^{-\lambda t_f}}{t_f \lambda} - \frac{\lambda e^{-(\lambda+r)t_f}}{t_f (\lambda + r)^2} \right] \quad \text{C3}
\]

- **Discrete** \(g(t)\):

\[
C = \frac{\alpha q}{\lambda + r} \left[ \frac{r}{1-e^{-\lambda t_f}} - \left( \frac{\lambda e^{-(\lambda+r)t_f}}{1-e^{-(\lambda+r)t_f}} \right) \left( \frac{r}{\lambda + r} \right)^{\lambda+r} \right] \quad \text{C4}
\]
3.4 Capacity of limited priority

- Circulating vehicle has to adjust headway to allow the entry vehicle into the intersection when entry vehicle merges. (limited priority merge) (Troutbeck, Kako, 1999; Troutbeck, 1998, 2002, Bunker, Troutbeck, 2003).

- In the exchange of priority, entry vehicle merges forcibly.

\[
C = \frac{\alpha f_L q e^{-\lambda(t_c-t_m)}}{1-e^{-\lambda t_f}}, t_m < t_c < t_f + t_m \quad \text{if} \quad t_c \geq t_f + t_m, f_L = 1
\]

\[
f_L = \frac{1-e^{-\lambda t_f}}{1-e^{-\lambda(t_c-t_m)}} - \lambda(t_c-t_f-t_m)e^{-\lambda(t_c-t_m)}
\]

When the circulating vehicles haven’t delay, the conditions of limited priority and exchange of priority are identical.
3.5 Comparison of some methods

C₁ is linear, others are nonlinear and the descending order of calculated values is C₃, C₂, C₄.
Three capacities are smaller than C₁ when q is a smaller value, relation is opposite when q is a larger value.

Gaps larger than average critical gap will be rejected sometimes under low flow rate in major stream, gaps smaller than average critical gap are probably accepted under high flow rate.
So minor stream vehicles often force into the intersection when the major stream is bunched.
4 Some Conclusions

- New equations of critical gap and capacity have reasonable results in accord with some classical methods.
- The equation (5) and (13) of critical gap was deduced based on the gap acceptance theory, however most of the equations are the experiential method.
- The equation (5) can be directly plugged into the capacity equations and simplify some complex capacity equations.
- The new equations of capacity have more accurate results under some conditions, for example, the low capacity is closer to the reality than linear regression under low flow rate in major stream.
Regular Conclusions

- It is more realistic that drivers are inconsistent and nonhomogeneous, but the calculation error is only a few percent. (Troutbeck, Brilon, 1997);
- The circulating headway is always regarded as negative exponential distribution, and M3 distribution should be emphasized;
- Methods such as Ashworth, Logit, Maximum Likelihood and so on, aren't related with gap acceptance theory;
- The method based on stochastic critical gap is more realistic with operation in roundabouts, but the corresponding equations are too complicated to calculate. Simplification is needed to apply in the field.
Application of Capacity

- HCM(2000) are used Capacity model of discrete function, M1 distribution, and fixed parameters of gap acceptance which are based on limited quantity survey data. Inaccurate for single-lane roundabout when circulating flow rate is up to 1200pcu/h.
- HCM(2010) used capacity model of continuous function, M1 distribution, fixed parameters of gap acceptance and built practical equations with different numbers of circulating lanes and entry lanes.
- aaSIDRA used capacity model of continuous function, M3 distribution, and parameters of gap acceptance which are based on Australian roundabout data. Capacity is related with lane use, OD matrix of flow rate, queue in entry and the proportion of free vehicles.
- Linear regression model in England originated from the traditional roundabout with large diameter. Capacity is related with circulating flow rate and geometric parameter.
- HCM model and linear regression model are applied successfully and easily.
Reference


Thank you