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SCHEDULING WORK ZONES IN MULTI-MODAL NETWORKS
Phase I: Scheduling Work Zones in Transportation Service Networks

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EXECUTIVE SUMMARY

The purpose of this project is to study the optimal scheduling of work zones so that they have minimum negative impact (e.g., travel delay, gas consumption, accidents, etc.) on transport service vehicle flows. In this project, a mixed integer linear programming model is developed to schedule work zones in transportation service (i.e., trucking service) networks. The model schedules lane closures of links that need maintenance in a transportation network. When some lanes of a link are closed, the available capacity of that link is reduced. In the assumed scenarios, based on the available capacities on the links, given origin-destination (OD) flow demands are provided system optimal routing through the network to achieve total minimum flow cost for all the OD pairs. The link flow cost function is piece-wise linear such that regular flow cost is incurred for all the units flowing through the link at free flow while extra congestion cost is incurred for the units exceeding the link’s nominal capacity. The goal is to schedule the work zones, that is, the corresponding lane closures, so that all maintenance work can be completed before a given completion date while the total flow cost over the project period is minimized. An innovative randomized fix-and-optimize (RFO) heuristic is developed to solve the problem efficiently. Various networks are tested for the performance comparison between CPLEX and RFO. It is concluded that the RFO heuristic is able to obtain optimal or near-optimal solutions with much less time than CPLEX. In Phase 2 of the project planned in the next period, Scheduling Work Zones in Commuter Transportation Networks, commuters from their origins to their destinations are included in the work zone scheduling problem.

*Keywords:* Transportation Service Network, Work Zone Scheduling; Mixed Integer Linear Programming (MIP); Randomized Fix-and-optimize Heuristic (RFO)

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1. Introduction

The repair and maintenance of road network results in “work zones”, where some lane segments of a link are out of commission for a predicted period of time until the work is completed. Temporary link capacity reductions caused by lane closure can result in significant delays of commuters and transport services. Americans lose 3.7 billion hours and 2.3 billion gallons of fuel every year sitting in traffic jams. Work zones are estimated to cause about 10% of overall congestion which translates into annual fuel loss of over 700 million US dollars (FHWA, 2013).

Road construction companies and transportation management agencies do a reasonable job of coordinating work zone activities after the work zone is initiated through appropriate scheduling and staging of day-to-day and week-to-week operations so that the overall cost is contained, while safety and traffic congestion is not overly affected during peak periods. While a single, or few widely scattered concurrent work zones, will not have a large effect upon travel patterns, several work zones that are spatially and temporally close together, and which affect large flows of traffic, may result in traffic patterns that are both costly to the travelers and vehicle-based services, resulting in significant negative environmental and safety consequences. In 2010, there were 87,606 crashes in work zones and 526 of these crashes were fatal (FHWA, 2013). While large trucks accounted for only 4% of all registered vehicles in the United States, 27% of work zone fatal crashes involved at least one large truck (FHWA, 2013). Through proper scheduling of work zones with respect to the spatial locations in the network and the time periods of the work zones, a reduction of negative impacts is expected.

In this project, we consider a network that provides transport logistics services (e.g., freight and parcel delivery) from origins to destinations, where the overall objective is to minimize system transportation costs. The traveling cost of a link is treated as the cost in general sense, which can be interpreted as combinations or functions of travel time, monetary cost, and road unsafety. Origin-Destination (OD) flows of vehicles are reactive to the network topology changes resulted from temporary lane closures, which means the routes chosen by each vehicle could change in response to work zone activities to minimize their flow costs. Since this project focuses particularly on trucks and other delivery vehicles, we use the term “trucks” instead of the generic “vehicles” term. The model developed finds the optimal lane closure schedule that has the minimum negative impact (e.g. gas usage, congestion) on the trucks’ flows in the network.

2. Related Literature Review

Work zone planning is a challenging task since there are multiple parties involved and more than many factors need to be taken into consideration. Bayraktar and Hastak (2009) reviewed the factors impacting the success of work zone projects. They modeled the relationships between the goals of the project...
stakeholders and public satisfaction of the project using Bayesian belief networks. The model was aimed to assist highway agencies in developing suitable contracting strategies considering 52 interrelated factors impacting the success of work zone projects, which were grouped into four categories (contract characteristics, motorist issues, public issues, and resource issues). Despite the comprehensive list of factors taken into account, the model can only help prepare bids and not help to actually schedule the work zones.

Most of the literature related to the problem of this project can be grouped into three categories. The first category includes research that investigates the long term network rehabilitation planning problem with the objective of maintaining the roads in good condition with least cost in different aspects. For example, Chu and Chen (2012) developed a bi-level hybrid dynamic model in which the upper level problem decides the optimal threshold for each road that triggers maintenance action and the lower level problem solves the user equilibrium problem. These two levels of problems are connected by the road deterioration function which models the effects of traffic loads on a road and the impacts of road roughness on users’ traveling cost. This type of research considers network-wide maintenance planning over a relatively long period of time (a year or longer). By assuming the project period is much shorter than the planning horizon, they omitted the impact of temporary link capacity reductions on traffic flow caused by the maintenance work. However, this assumption is not always reasonable especially for the maintenance work like resurfacing sets of links which would take months or longer. When the length of project period is comparable to the planning horizon, it is necessary to consider the effect of temporary link capacity reductions and to schedule the work zones in the way that minimizes the negative impacts on traffic flows.

Research in the second category focuses on developing operational strategies for work zone scheduling on a highway segment or a local arterial. Some research in this category has studied the short term work zone scheduling with time horizons less than a day. This research focuses on optimizing the workzone planning of a single link but does not consider the impact of diverting traffic and possible resulted from workzones to other links that are connected to or close to the focusing link; see e.g., works of Meng and Weng (2013), Tang and Chien (2008), and Jiang and Adeli (2003). However, in reality, as long as traffic congestion exists and there are alternative routes available, some portion of the traffic will divert to other routes which will affect the traffic on those alternative routes. Chien and Tang (2014) proposed a genetic algorithm to optimize the work zone length and start time in a day of the maintenance work on a highway stretch. The optimal schedule minimizes the total cost to the agencies conducting the maintenance plus the cost to the road users. Even though the temporary link capacity reductions, and resulting increased road user cost, and possible traffic diversion, were modeled, only one alternative route for the diverted traffic was considered. Often there are more than two lanes for some segments of highway, but Chien and Tang (2014) did not explicitly explore different lane closing scenarios. Schroeder and Roupail (2010) compared different lane closure scenarios and discussed the operational impacts of freeway work zones on traffic.
Their approach can only compare every limited number of scenarios since each scenario requires extensive analysis. Summarizing, the research in this category focuses on scheduling work zones on single links and has very limited or no consideration on the impact of traffic diversion resulting from multiple link capacity reductions.

The third category consists research that studied the scheduling of network expansion projects. These research specifically considered the flow pattern changes caused by the increase of link capacities or the addition of new links over the planning time horizon. This research topic is closely related to the network design problem, which selects among a set of candidate links to be added to a network with budget constraints, so as to achieve lowest total cost at users’ equilibrium state or system optimum. It is an extension of the network design problem since the addition of the chosen links need to be scheduled, and possible traffic flow pattern changes need to be evaluated after the addition of each link. Fontaine and Minner (2014) developed a mixed-integer programming model to select and schedule network expansion projects with minimum total project cost and system optimum flow cost, and solved it using Bender’s decomposition. Bagloee and Asadi (2015) presumed the set of network expansion projects were given and only one of these projects could be worked on at a time, and studied the network expansion scheduling problem as a traveling sales man problem to determine the optimal sequence of the expansion projects. The inter-dependency of the expansion projects was evaluated using the artificial neural network model, so that the “cost” of “moving” from one expansion project to another could be computed. Gao et al. (2011) combined the problems of road maintenance and road expansion planning, and developed a mixed-integer, nonlinear, bi-level model that scheduled the repair or expansion of every road with budget constraints. In the model proposed, the road capacity increase after maintenance and expansion were considered, and the road degradation process was modeled. General Bender’s decomposition method was applied to obtain the optimal maintenance and expansion schedule that gave the minimum total users’ cost at equilibrium state. Although literature reviewed in this category modeled the capacity increase after the maintenance or expansion, they did not consider the link capacity reductions during the time period when these activities were being performed.

Only a handful of works considered the impact on traffic over the network due to multiple work zones and they comprised the fourth category. Orabi and El-Rayes (2012) developed a complex model with three genetic algorithm based modules – scheduling, network performance, and user savings, to select and prioritize rehabilitation projects, subject to budget constraints. Lee (2009) proposed a work zone scheduling model which considered the routing-changing behavior of road users. The schedule was optimized with an ant colony algorithm, where the users’ equilibrium under each schedule scenario was obtained through simulations using VISSIM software. Hosseininasab and Shetab-Boushehri (2015) studied the work zone scheduling problem as a time-dependent network design problem. They formulated the problem as bi-level
programming models, and used genetic algorithm to obtain the link maintenance schedule that gave the minimum total traveling cost at equilibria over the planning time horizon. All the three of Orabi and El-Rayes (2012), Lee (2009) and Hosseininasab and Shetab-Boushehri (2015) did not explicitly discuss partial link capacity reductions resulting from work zones. Zheng et al. (2014) assumed the link capacity would reduce by 50% in their decision model developed. However, a link might have more than two lanes and it is not always true or optimal to close half of the lanes at a time for maintenance. Ma et al. (2004) developed a hybrid simulation methodology with genetic algorithm to schedule multiple lane closures with minimum total traffic delay of the network. However, the flexible lane-level maintenance scheduling required high computation effort for the solution approach proposed in Ma et al. (2004). For a problem instance of scheduling the maintenance of 20 lanes, it took more than 120 hours.

In this project, a mixed integer linear programming model is developed to schedule work zones in the perspective of networks that are used by trucks fulfilling transportation services. A randomized fix-and-optimize (RFO) heuristic is developed to solve the problem efficiently. In Section 3 provides a complete description of the work zone scheduling model for transportation service networks (WZS-TS) and its computational implementation. Section 4 describes the details of the heuristic developed and illustrates its performance by comparing it with an exact CPLEX-based approach on various test cases. Section 5 provides conclusions and gives some directions of possible future research.

3. Work Zone Scheduling in Transportation Service Networks (WZS-TS) Model

3.1 Piecewise Linear Cost Structure

In transportation service networks, linear flow cost structure is commonly used for minimum flow cost problems, where we set the cost of travelling on a link linear with respect to the total flow on that link when the total flow is smaller than or equal to the available nominal capacity of the link. In applications where the demand on a link is more than the available capacity, the excess flow is either detoured or given a very high cost for using the link thereby softening the hard capacity constraint. In this project we will use the latter approach by modeling the cost as a piece-wise linear cost to approximate the traffic condition aggravation effects in transportation networks. With the piece-wise linear cost functions the work zone scheduling model, developed later, can be solved by commercial solvers like CPLEX, the performance of which can be used to compare with the new heuristic developed later in the project.

In the workzone scheduling model, it is assumed that there are Origin-Destination (OD) flow demands of trucks every time period (e.g., peak period of a day). Each truck can choose its own route to minimize its travel cost and is treated as a unit of flow. When a link is under maintenance, one or more lanes are closed, inducing the temporary link capacity reductions, and thus the link cannot fully serve the flows
That is likely to cause the current flow on the link to exceed the available nominal capacity, incurring the expensive extra flow cost. Suppose a link has three lanes and all three lanes have the same “flow capacity” \( u \), Figure 1 below illustrates the relation between the flow units and flow cost for different lane closure situations:

![Flow Cost Curve](image)

**FIGURE 1: Three-Lane Link Flow Cost Curve.**

When two lanes are closed for maintenance, the available capacity of the link is \( u \). If the units of flows on the link is more than \( u \), then the extra flow cost will be incurred. This is why the slope of the cost curve is much steeper when the flow units is more than \( u \) for the case of two-lane closure. Same cost curve pattern can be seen in the cases of no-lane closure and one-lane closure. When some of the lanes in a link is closed for maintenance, some of the flow that is originally on this link could divert to other paths and links to reach the destination with lower total, that is, the network flows are reactive to the maintenance schedules.

### 3.2 Model Formulation

The objective of the model is to schedule the lane closures so that all links that need maintenance are repaired before a given completion date for the whole network, while the total flow cost for all the OD pairs, which includes regular flow cost and extra congestion flow cost, is minimized over the project period. Denote \( c_i \) as the regular unit flow cost of link \( i \), \( y_{ikt} \) as the flow units of OD pair \( k \) that flow through link \( i \) on day \( t \), and \( z_{it} \) as the difference between flow units of all the OD pairs that flow through link \( i \) and the available capacity of link \( i \) on day \( t \), the objective function (1) is formulated as

\[
\min \sum_{i \in E} \left\{ \sum_{t=1}^{T} [c_i \ast (\sum_{k \in OD} y_{ikt}) + z_{it} \rho c_i] \right\},
\]

where \( E \) is the set of links, \( OD \) is the set of OD demand, and \( T \) is the common completion date of all the maintenance work. \( \rho \) is the congestion flow cost multiplier which makes the extra unit flow cost \( \rho c_i \) much larger than the regular unit flow cost \( c_i \). Note that \( z_{it} \) is non-negative in the sense.
that it will have positive value only when the total flow units on link \( i \) exceed the available capacity and it will be zero otherwise.

Binary variables \( s_{imt} \) are introduced to indicate whether the repair of the \( m^{th} \) lane of link \( i \) starts on day \( t \), and \( s_{imt} = 1 \) if it is. The WZS-TS model assumes once a lane is closed for repair, it cannot open to serve the flows until its repair is completed. Hence we have the constraints (2): \( \sum_{t=1}^{T} s_{imt} = 1 \) for \( \forall i \in R \) and \( \forall m \in [1, n_i] \), where \( R \) is the set of links that need repair and \( n_i \) is the number of lanes in link \( i \). This set of constraints force every lane of all the links that need repair to have one and only one repair start date.

To indicate whether \( m^{th} \) lane of link \( i \) is closed for maintenance on day \( t \), binary variables \( x_{imt} \) are added to the model. \( x_{imt} \) equal to 1 if the \( m^{th} \) lane of link \( i \) is closed for maintenance on day \( t \). Let \( p_i \) be the number of days needed to repair a lane of link \( i \), we formulate the constraints (3) \( \sum_{t=1}^{T} x_{imt} = p_i \) for \( \forall i \in R \) and \( \forall m \in [1, n_i] \) to ensure the repair on all the links be completed by the common completion date \( T \). Since each lane of the links needing maintenance have one and only one repair start date and the number of days needed to repair a lane is given, whether a lane is closed or not on a day is determined once the repair start date of that lane is determined. And thus, we develop the set of constraints (4) \( x_{imt} = \sum_{a=max(t-p_i+1,1)}^{\min(T,t)} s_{ima} \) for \( \forall i \in R, \forall t \in T \) and \( \forall m \in [1, n_i] \) to make sure that once a lane is closed for repair, it will not open to serve the flows until the repair work on this lane is finished and that it will be open on other dates. Constraints (5) \( \sum_{t=1}^{T} s_{imt} = 0 \) for \( \forall i \in R, \forall m \in [1, n_i] \) and (6) \( \sum_{t=1}^{T} x_{imt} = 0 \) for \( \forall i \in R \) and \( \forall m \in [1, n_i] \) are added to the model so that all the lanes of links that do not need repair will not have maintenance start date and will be open to serve the flows throughout the project period.

For each OD pair on each day, flow conservation constraints, consisting of three groups, are needed. The first group of constraints makes sure the total incoming flow units minus the total outgoing flow units equal to the OD demand for the origin node of the OD pair. Let \( D_k \) be the demand of OD pair \( k \), the first part is formulated as (7) \( D_k = \sum_{i} \{ i \in E_i = OD_k^-, i \in E \} y_{ikt} - \sum_{j} \{ j \in E_j^+, j \in E \} y_{jkt} \) for \( \forall k \in OD, \forall t \in [1, T] \), where \( OD_k^- \) is the origin node of OD pair \( k \), \( E_i^- \) is the head node of link \( i \) and \( E_j^+ \) is the tail node of link \( j \). The second group ensures the total outgoing flow units equal to the demand of OD pair \( k \) for its destination node and is formulated as (8) \( D_k = \sum_{i} \{ i \in E_i^+ = OD_K^+, i \in E \} y_{ikt} - \sum_{j} \{ j \in E_j^- = OD_k^-, j \in E \} y_{jkt} \) for \( \forall k \in OD, \forall t \in [1, T] \), where \( OD_k^+ \) is the destination node of OD pair \( k \), \( E_i^+ \) is the tail node of link \( i \) and \( E_j^- \) is the head node of link \( j \). For the rest of the nodes, other than origin and destination nodes of OD pair \( k \), the total incoming flows on the node from the origin of OD pair \( k \) should equal to the total outgoing flows from the node to the destination of the OD pair \( k \). This is the third group
of the flow conservation constraints and it is formulated as (9) \( \sum_{(i \in E, l \in E)} y_{ikt} = \sum_{(j \in E, l \in E)} y_{jkt} \) for \( \forall l \in N, \forall t \in [1, T], \forall k \in \{k: OD_{ik} \neq l\} \cap \{k : OD_{ik}^+ \neq l\} \), where \( N \) is the set of nodes in the network.

In addition, binary variables \( v_{imt} \) are introduced to calculate the increased lane capacities and \( v_{imt} \) equals to 1 if lane \( m \) of link \( i \) is repaired before day \( t \), since it is obvious that when a segment of road is repaired, the road condition should be improved and the capacity should increase. Constraints (10) \( v_{imt} = \sum_{a=1}^{a=t-p_i} s_{ima} \), for \( \forall i \in R, \forall m \in [1, n_i] \) and \( \forall t \in [p_i + 1, T] \) determine the values of \( v_{imt} \) by values of \( s_{ima} \). In the constraints, the date ranges from \( p_i + 1 \) to \( T \) since the lane will be repaired and open to serve the flows on day \( p_i + 1 \) the earliest, because even if the maintenance starts on day 1, it would take \( p_i \) days to complete the repair work for this lane. Constraints (11) \( v_{imt} = 0 \), for \( \forall i \in R, \forall m \in [1, n_i] \) and \( \forall t \in [1, p_i] \) make sure each lane of the links that need maintenance stay in the status of not repaired in the first \( p_i \) days. And constraints (12) \( v_{imt} = 0 \), for \( \forall i \in R, \forall m \in [1, n_i] \) and \( \forall t \in [1, T] \) force lanes of links that do not need repair stay in the status of not repaired throughout the project period.

Let \( \theta \) be the percentage increase in lane capacity after the lane is repaired, and let \( u_i \) be the capacity of a lane of link \( i \), the available capacity of link \( i \) on day \( t \) is \( (n_i - \sum_{m=1}^{n_i} x_{imt} + \sum_{m=1}^{n_i} \theta v_{imt})u_i \). Hence the values of \( z_{it} \) are determined by constraints (13) \( \sum_{k \in OD} y_{ikt} - (n_i - \sum_{m=1}^{n_i} x_{imt} + \sum_{m=1}^{n_i} \theta v_{imt})u_i \leq z_{it} \) and \( z_{it} \geq 0 \) for \( \forall i \in E \) and \( \forall t \in [1, T] \), where \( \sum_{k \in OD} y_{ikt} \) are the total flow units from all OD pairs on link \( i \) on day \( t \).

Because of the introduction of \( z_{it} \), flows can exceed the available capacity. Hence it is needed to make sure there won’t be flows on links with all lanes closed for maintenance, that is, entirely closed links cannot serve any flow. For this reason, the set of variables \( w_{it} \) are added into the model, the values of which equal to 1 if all the lanes of link \( i \) are closed on day \( t \). Constraints (14) \( 1 - w_{it} \leq n_i - \sum_{m=1}^{n_i} x_{imt} \) for \( \forall i \in R \) and \( \forall t \in [1, T] \) make sure \( w_{it} \) equal to 1 when all the lanes of link \( i \) are closed on day \( t \), and constraints (15) \( n_i - \sum_{m=1}^{n_i} x_{imt} \leq n_i(1 - w_{it}) \) for \( \forall i \in R \) and \( \forall t \in [1, T] \) force \( w_{it} \) to be 0 if at least one lane of link \( i \) is open on day \( t \). Finally, constraints (16) \( \sum_{k \in OD} y_{ikt} \leq \sum_{k \in OD} D_k (1 - w_{it}) \) for \( \forall i \in R \) and \( \forall t \in [1, T] \) prevent links with all lanes closed from serving flows.

The sets, parameters, and variables mentioned in the model description above are summarized in Table 1 below:

**TABLE 1: Notations**

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N )</td>
<td>Node set of the network</td>
</tr>
<tr>
<td>( E )</td>
<td>The set of existing links in the network</td>
</tr>
</tbody>
</table>

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The complete model of work zone scheduling in transportation service networks (WZS-TS) is presented in Figure 2, where the number of each expression matches the bracketed numbers in the model description earlier in this subsection.

\[
\min \sum_{i \in E} \{ \sum_{t=1}^{T} [c_i \ast (\sum_{k \in OD} y_{ikt}) + z_{it} \rho c_i] \}
\]

(1)

\[
\sum_{t=1}^{T} s_{imt} = 1, \quad \forall i \in R, \forall m \in [1, n_i]
\]

(2)

\[
\sum_{t=1}^{T} x_{imt} = P_i, \quad \forall i \in R, \forall m \in [1, n_i]
\]

(3)

\[
x_{imt} = \sum_{a=max(t-p+1,1)}^{T} s_{ima}, \quad \forall i \in R, \forall t \in T, \forall m \in [1, n_i]
\]

(4)

\[
\sum_{t=1}^{T} s_{imt} = 0, \quad \forall i \notin R, \forall m \in [1, n_i]
\]

(5)

\[
\sum_{t=1}^{T} x_{imt} = 0, \quad \forall i \notin R, \forall m \in [1, n_i]
\]

(6)

\[
D_k = \sum_{i \in E^+_k} y_{ikt} - \sum_{j \in E^-_k} y_{jkt}, \quad \forall k \in OD, \forall t \in [1, T]
\]

(7)

\[
D_k = \sum_{i \in E^+_k} y_{ikt} - \sum_{j \in E^-_k} y_{jkt}, \quad \forall k \in OD, \forall t \in [1, T]
\]

(8)

\[
\sum_{i \in E^+_l} y_{ikt} = \sum_{j \in E^-_l} y_{jkt}, \quad \forall l \in N, \forall t \in [1, T], \forall k \in \{k: OD_k^-=l\} \cap \{k: OD_k^+=l\}
\]

(9)
\[ v_{imt} = \sum_{a=1}^{\alpha} t - p_i s_{ima}, \quad \forall i \in R, \forall m \in [1, n_i], \forall t \in [p_i + 1, T] \]  
\[ v_{imt} = 0, \quad \forall i \in R, \forall m \in [1, n_i], \forall t \in [1, p_i] \]  
\[ v_{imt} = 0, \quad \forall i \in R, \forall m \in [1, n_i], \forall t \in [1, T] \]  
\[ \sum_{k \in OD} y_{ikt} - (n_i - \sum_{m=1}^{n_i} x_{imt} + \sum_{m=1}^{n_i} \theta v_{imt}) u_i \leq z_{it}, \quad \forall i \in E, \forall t \in [1, T] \]  
\[ 1 - w_{it} \leq n_i - \sum_{m=1}^{n_i} x_{imt}, \quad \forall i \in R, \forall t \in [1, T] \]  
\[ n_i - \sum_{m=1}^{n_i} x_{imt} \leq n_i (1 - w_{it}), \quad \forall i \in R, \forall t \in [1, T] \]  
\[ \sum_{k \in OD} y_{ikt} \leq \sum_{k \in OD} D_k (1 - w_{it}). \quad \forall i \in R, \forall t \in [1, T] \]  
\[ w_{it} = 0, \quad \forall i \notin R, \forall t \in [1, T] \]  
\[ w_{it} \in \{0, 1\}, \quad \forall i \in E, \forall t \in [1, T] \]  
\[ s_{imt}, x_{imt}, v_{imt} \in \{0, 1\}, \quad \forall i \in E, \forall m \in [1, n_i], \forall t \in [1, T] \]  
\[ z_{it} \geq 0, \quad \forall i \in E, \forall t \in [1, T] \]  
\[ y_{ikt} \geq 0, \quad \forall i \in E, \forall k \in OD, \forall t \in [1, T] \]

**FIGURE 2: Work Zone Scheduling in Transportation Service Networks (WZS-TS) Model.**

The WZS-TS model possesses the features of both scheduling models and multi-commodity flows models. For each feasible schedule on each day, there is a multi-commodity flows problem over the network based on links’ available capacities after the scheduled lane closures. As shown in Figure 3, with lane closure schedules fixed, variables \( x_{imt}, v_{imt}, \) and \( w_{it} \) become parameters \( x_{imt}^0, v_{imt}^0, \) and \( w_{it}^0 \) that define the links’ available capacities for each day, which is calculated as \( (n_i - \sum_{m=1}^{n_i} x_{imt}^0 + \sum_{m=1}^{n_i} \theta v_{imt}^0) u_i \) in Constraint (13'). And the remaining problem is a multi-commodity flows problem for each day over the planning time horizon. In the multi-commodity flows subproblem of WZS-TS, the available link capacity can be exceeded with a very high extra flow cost. This is different from the traditional multi-commodity flows problem, where link capacity constraints are hard constraints and the total amount of flows on the link has to be less than or equal to the link’s capacity.

\[
\min \sum_{i \in E} \left\{ \sum_{t=1}^{T} [c_i \ast (\sum_{k \in OD} y_{ikt}) + z_{it} \rho c_i] \right\} \quad (1)
\]

\[
D_k = \sum_{(i \in E^- = OD_k, j \in E)} y_{ikt} - \sum_{(j \in E^+ = OD_k, i \in E)} y_{jk t}, \quad \forall k \in OD, \forall t \in [1, T] \quad (7')
\]

\[
D_k = \sum_{(i \in E^+ = OD_k, j \in E)} y_{ikt} - \sum_{(j \in E^- = OD_k, i \in E)} y_{jk t}, \quad \forall k \in OD, \forall t \in [1, T] \quad (8')
\]

\[
\sum_{(i \in E^- = l, j \in E)} y_{ikt} = \sum_{(j \in E^+ = l, j \in E)} y_{jk t}, \quad \forall l \in N, \forall t \in [1, T], \forall k \in \{k: OD_k^- \neq l\} \cap \{k: OD_k^+ \neq l\} \quad (9')
\]

\[
\sum_{k \in OD} y_{ikt} - (n_i - \sum_{m=1}^{n_i} x_{imt}^0 + \sum_{m=1}^{n_i} \theta v_{imt}^0) u_i \leq z_{it}, \quad \forall i \in E, \forall t \in [1, T] \quad (13')
\]

\[
\sum_{k \in OD} y_{ikt} \leq \sum_{k \in OD} D_k (1 - w_{it}^0), \quad \forall i \in R, \forall t \in [1, T] \quad (16')
\]

\[ z_{it} \geq 0, \quad \forall i \in E, \forall t \in [1, T] \quad (20')
\]

\[ y_{ikt} \geq 0, \quad \forall i \in E, \forall k \in OD, \forall t \in [1, T] \quad (21') \]
3.3 Computational Implementation

Even et al. (1975) proved that the decision version of the multi-commodity flow problem is NP-Complete even for only two commodities and unit capacities. By setting $T = 1$ and $R = \emptyset$, the problem discussed in this project is converted to a multi-commodity flow problem, which proves that this problem can be reduced from multi-commodity flow problem in polynomial time. Thus the WZS-TS problem is strongly NP-Hard.

The WZS-TS model is programmed in C++ with IBM® ILOG® CPLEX® Concert Technology. Some preliminary experiments are conducted and the results indicate the non-polynomial nature of the problem. To give an example, with a computer of 3.7 GHz quad-core CPU and 24.0 GB memory, it takes 0.51 second to solve a problem instance of 4 nodes, 12 links, 30 lanes, 12 OD pairs, 25 days of project completion period with all links needing repair; while to solve a problem instance of 16 nodes, 48 links, 108 lanes, 16 OD pairs, 27 days of project period with 50% of the links needing maintenance, CPLEX still has a 32% optimality gap after 14 hours of computation. Therefore, it is clear an efficient heuristic to solve the problem quickly with satisfactory accuracy is needed.

4. Solution Approach

4.1 Randomized Fix-and-Optimize (RFO) Heuristic

There are two layers of problems that constitute the problem of work zone scheduling in transportation service networks. The upper layer is the scheduling problem which decides the repair start date for each lane of the links that need maintenance. The lower layer is a series of multi-commodity flow problems based on the available capacities of links on each day, which is determined by the current lane closures. Once the schedule is set, solving the multi-commodity flow problems for each day is a relatively easy problem since the flow variables are all continuous variables. And thus the solution approach proposed in this project focuses on the upper layer of obtaining good work zone schedules.

To motivate the heuristic, suppose at a point in the algorithmic process we obtain a feasible schedule that has some aspects similar to the optimal schedule. For example, Figure 4 on the next page gives a comparison between the Gantt charts of the optimal schedule and one of the feasible schedules obtained for a small test network of 4 nodes, 12 links and 12 OD pairs. The vertical axis shows the lanes of links that need maintenance and the horizontal axis shows the date during the project period. Each bar represents the time period when a lane is closed for maintenance and cannot be used to serve the OD flows. For example, in the optimal schedule, Lane 1 of Link 2 is closed on Day 1 and will be reopen on Day 8, and Lane 2 of Link 2 will be closed from Day 7 to Day 13. Hence this two-lane link will have one lane available from
Day 1 to Day 6 and from Day 8 to Day 13. On Day 7 Link 2 is not available to serve any flows since both of the two lanes are closed.

From the Gantt charts we can see that the feasible schedule has lane closures of Link 1, 3, 7, and 12 different from the optimal schedule. If we only optimize the lane closure schedules of these four links and fix the schedules of all the other links, the problem size will be much smaller and the time needed to solve the problem instance will reduce dramatically since there are much fewer integer variables to go through in the branch-and-bound process performed by solvers like CPLEX. This observation leads to the adoption of the fix-and-optimize heuristic as the core of the solution approach.

<table>
<thead>
<tr>
<th>Optimal Schedule</th>
<th>Some Feasible Schedule</th>
</tr>
</thead>
</table>

![Schedule Comparison](image)

**FIGURE 4: Schedule Comparison.**
The fix-and-optimize heuristic was first introduced by Helber et al. (2010). It is an iterative optimization-based heuristic developed to solve the multi-level capacitated lot sizing problem which is a mixed integer program. The basic process of the fix-and-optimize heuristic is to partition the integer variables into subsets, based on an initial solution, and then optimize the values of a subset of integer variables together with all continuous variables while the values of the other integer variables in other subsets are fixed (this is called a subproblem of the fix-and-optimize procedure). If the new objective function value is better than current best objective value, then the current candidate optimal values are updated; iterate this process for other subsets of variables until a specified stopping criteria is met. The integer variables were decomposed into subsets based on the descending order on cost of each product in the lot-sizing problem, since usually a quite reasonable schedule was found after the first round of the product-oriented decomposition.

In the problem of scheduling work zones in transportation service networks, the relation among work zones is more complex than that among products in the capacitated lot-sizing problem. Products just compete with each other for resources (machine hours) in the capacitated lot-sizing problem. On the other hand, in the WZS-TS problem there are no resource constraints that work zones compete for, but instead the work zones affect the capacity of the network to serve the OD demands. Therefore, only the schedules that consider all or many work zones will have the lowest increase in total flow cost, because OD demands happen over the whole network and each OD pair has network-wide minimum cost routing. This means applying fix-and-optimize heuristic with small subsets of work zones (one or two links) will hardly find satisfactory schedules since it only considers the maintenance of a few links at a time.

However, if the size of the work zone subsets is large, the size of each fix-and-optimize subproblem will also be large and it would take long time to solve. To mitigate the conflict between solution quality and solving time length, we developed the fix-and-optimize procedure with varying subset sizes and used a truncated branch-and-bound method.

Initially, CPLEX tries to solve the entire problem within a given time limit (e.g. 60 seconds). If the problem is solved optimally, then the optimal schedule will be output and the program will terminate. If the problem is not solved optimally, the best feasible schedule obtained so far will be stored and used as the initial feasible solution for the fix-and-optimize procedure. A feasible schedule should be able to both complete all the maintenance work before the specified completion date and make sure each OD demands can be met.

The randomized fix-and-optimize (RFO) iteration starts with randomly dividing links that need maintenance into two subsets and solving each fix-and-optimize subproblem (FO subproblem) with a specified time limit. A RFO iteration is finished when the schedules of all the generated subsets of links are
optimized. The RFO will be performed for a preset number of iterations and if any of the FO subproblems is not solved within the time limit in the last iteration, the RFO will enter a new stage where the number of subsets which the links to repair are randomly divide into is three. The RFO proceeds similarly in stages with more subsets of links and each RFO iteration is performed the same way as it is in the initial stage when there are only two subsets.

The reason of randomly grouping links that need maintenance into subsets is because we do not know the set of links with schedules that are different from the optimal schedule since we do not have the optimal schedule. Also, consideration of various OD demand patterns, and flows being reactive to network capacity changes, makes it formidable to pin-point the links that can have better schedule through classical network flows optimization models. Hence random grouping is applied to explore various combinations of links for better schedules. Both the decomposition of the links based on the required number of days to repair and decomposition based on links’ unit flow cost were tested, but both of them had inferior performance compared to the random grouping approach. Through the iterative randomized fix-and-optimize process, the work zone schedule changes gradually towards the optimal schedule.

4.2 Parameters Affecting the Performance of RFO

The randomized fix-and-optimize heuristic has two layers of computation procedures. The first layer randomly decomposes the links that need maintenance into a specific number of subsets and the second layer optimizes the repair schedules of each link subset with the schedules of links in other subsets fixed (FO subproblem) within a specified time limit. Hence the efficiency of RFO heuristic is mostly determined by two parameters: the number of iterations RFO performs for a specific number of groups which the links to repair are randomly partitioned, and the time limits for the initial attempt on solving the entire problem and for the attempts on each FO subproblem.

More RFO iterations means that the heuristic can solve FO subproblems for more combinations of links to repair for a specific subset size and is more likely to obtain better feasible solutions with objectives that are closer to the optimal solution. However, after a considerable amount of experimentation, we found that increasing the number of iterations does not effectively improve the solution quality. This is because there are too many possible combinations of links to repair for any specific subset size, and the chance is little that the links, which have schedules different from the optimal schedule, are in the same subset through random decomposition. Fewer subsets with more links in each subset can increase the chance of grouping together the links with repair schedules different from the optimal schedule. However, the time needed to find better schedules for each FO subproblem will be longer since now the FO subproblem has large number of integer variables. Thus, performing large number of iterations with fewer subsets with many links in one group will either result in poor solution quality with low time limit for each FO subproblem, or result in
very long solving time with high time limit for each FO subproblems. As default values, we set the number of iterations the same as the specified number of link groups (e.g. perform 2 RFO iterations when the number of groups is 2), and the numerical results in next section will show the RFO gives good feasible solutions within reasonable amount of time.

We also need the time limits for the initial attempt on solving the entire problem and for attempts on each FO subproblem. Problem instances with a few work zones have less integer variables, and is more likely to obtain a feasible solution that is close to the optimal solution (solution with less than 5% relative optimality gap) in a short time during the initial attempt to solve the entire problem. For each FO subproblem, if there is a feasible schedule that is better than the current best feasible schedule, the solver should be able to find it very quickly since the FO subproblem has even less integer variables. As long as a feasible schedule is found that is better than the current best feasible schedule, it can be used as the initial schedule for the next RFO iteration. Increasing the time limit in this case is pointless since a better schedule is already found and increased time will be wasted on improving the lower bound to prove the solution is optimal for the FO subproblem or the entire problem.

As the number of work zones increases, the dramatic increases in the number of combinations of integer variables complicates the branch-and-bound process substantially. This makes it nearly impossible to quickly obtain a feasible solution that is close to the optimal solution in the initial attempt on the entire problem. Improving the quality of initial feasible solution through increasing the time limit is not wise since it is very likely that the relative optimality gap is still larger than 5% after hours of calculation. With an initial feasible solution which is not close to the optimal solution to start the RFO process, it would also be challenging for the solver to find feasible solutions that are much better than the current best feasible solution found in a short time in the FO subproblem. Therefore, increasing the time limit on solving the FO subproblem will be much more effective in finding better solutions since the FO subproblem has much fewer integer variables. And thus, both the time limits on the initial attempt on the entire problem and on the attempts on each FO subproblem should be relatively higher to allow the solver to spend more time on searching for better feasible solutions.

The C++ code of the randomized fix-and-optimize heuristic (RFO) is attached in the appendix, and the detailed procedure of the RFO is shown below:

**Randomized Fix-and-optimize Heuristic**

1. Solve the entire problem with time limit $timeLimitSV$
   
   If optimal solution obtained, proceed to 4.
   
   Otherwise store the best feasible schedule and objective, and go to 2.

2. Set number of subsets $N = 2$

Pitu B. Mirchandani and Dening Peng
3. Randomly divide links to repair into $N$ groups

3.1. Fix $(v, s, x, w)$ for links in $N - 1$ groups, set $\text{LonSolTime} = 0$, set iteration number $\text{iter\_num} = 1$

3.2. Solve the FO subproblem with time limit $\text{timeLimitFO}$ for the subset $(n)$ of links the $(v, s, x, w)$ of which are not fixed

   If optimal solution is not obtained in $\text{timeLimitFO}$ proceed to 3.2.1.

   3.2.1. Store the current best feasible schedule and objective, and set $\text{LonSolTime} = 1$

   Otherwise directly proceed to 3.3.

3.3. If the objective obtained in current FO subproblem is lower than the best objective of the FO subproblems obtained so far ($\text{TotalCostFO}$), update the $\text{TotalCostFO}$ and the schedule of links in subset $n$

   Otherwise directly proceed to 3.4.

3.4. Check whether there are subsets of links of which the FO subproblems are not solved

   If there are, proceed to 3.4.1.

   3.4.1. Choose one of the subsets to be subset $n$ and go back to 3.1

   Otherwise proceed to 3.4.2.

   3.4.2. If $\text{TotalCostFO} < \text{TotalCost}$ (best objective overall), proceed to 3.4.2.1.

   3.4.2.1. Update the value of $\text{TotalCost}$ with the value of $\text{TotalCostFO}$, increase $\text{iter\_num}$ by 1, go back to 3.

   Otherwise proceed to 3.4.2.2.

   3.4.2.2. If $\text{iter\_num} < N$, proceed to 3.4.2.2.1.

   3.4.2.2.1. Increase $\text{iter\_num}$ by 1, go back to 3.

   Otherwise proceed to 3.4.2.2.2.

   3.4.2.2.2. If $\text{LonSolTime} = 1$, proceed to 3.4.2.2.2.1.

   3.4.2.2.2.1. If $\text{numLinpBat} > 3$, increase subsets number $N$ by 1, set iteration number 1, go back to 3.

   Otherwise proceed to 4.

4. Output the best schedule and flows obtained

4.3 Numerical Results

The randomized fix-and-optimize heuristic is tested on three representative networks: a radial network, a grid network, and the Sioux Falls network. For each network, the links that need maintenance are randomly selected based on the preset percentage of links to repair. Test cases of a network vary by the parameter $T$, which is the completion date for all the maintenance work. The extra flow cost multiplier $\rho$ is set to 10000 and the percentage of lane capacity increase after repair $\theta$ is set to 20% for all the test cases. The computer used to run these tests cases is the same computer mentioned in Section 3.3.

We begin the test on the heuristic designed with a radial network. Radial transportation network structure is commonly found in large cities with long history like London and Paris. The radial network tested is a small network with 6 nodes, 20 links and 20 OD pairs (shown in Figure 5) with 10 randomly selected links that need maintenance. This network has a total number of 30 work zones to be scheduled (since a link has multiple lanes and each lane is an independent work zone). Details of this radial network (i.e., number of lanes of each link, number of days required to repair a lane, lane capacity, unit flow cost,
and whether maintenance is required) and the OD demand are listed in the appendix. The time limits for solving the entire problem initially and for each FO subproblem are both 60 seconds. The performance comparison between solving the test cases by randomized fix-and-optimize heuristic (RFO) and solely by CPLEX is shown in Table 2.

FIGURE 5: Radial Network
TABLE 2: RFO VS CPLEX in Radial Network

<table>
<thead>
<tr>
<th>Completion Date (T)</th>
<th>Solving Time</th>
<th>Objective Value</th>
<th>Optimality Gap</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RFO</td>
<td>CPLEX</td>
<td>RFO</td>
</tr>
<tr>
<td>12</td>
<td>1.89 sec</td>
<td>1.89 sec</td>
<td>489892</td>
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<td>4.37 sec</td>
<td>404316</td>
</tr>
<tr>
<td>14</td>
<td>10.70 sec</td>
<td>10.70 sec</td>
<td>318741</td>
</tr>
<tr>
<td>15</td>
<td>1.52 min</td>
<td>29.75 min</td>
<td>233166</td>
</tr>
<tr>
<td>16</td>
<td>3.25 min</td>
<td>&gt;14.87 hr.</td>
<td>170591</td>
</tr>
<tr>
<td>17</td>
<td>5.8 min</td>
<td>&gt;40.82 hr.</td>
<td>101516</td>
</tr>
<tr>
<td>18</td>
<td>6.5 min</td>
<td>&gt;2.69 hr.</td>
<td>25645</td>
</tr>
<tr>
<td>19</td>
<td>6.52 min</td>
<td>&gt;2.54 hr.</td>
<td>19067</td>
</tr>
<tr>
<td>20</td>
<td>4.07 min</td>
<td>&gt;15.73 hr.</td>
<td>9888.26</td>
</tr>
<tr>
<td>26</td>
<td>4.62 sec</td>
<td>4.62 sec</td>
<td>623.34</td>
</tr>
<tr>
<td>36</td>
<td>49.79 sec</td>
<td>49.79 sec</td>
<td>856.62</td>
</tr>
<tr>
<td>46</td>
<td>1.88 min</td>
<td>1.07 hr.</td>
<td>1090.17</td>
</tr>
</tbody>
</table>

For the solving time of CPLEX that has “>”, it means CPLEX is not able to solve the test case optimally after a long time and the solving process is terminated manually with the best upper bound and lower bound obtained recorded. The upper bound is the objective value of the best feasible solution obtained at the time of terminating the solving process. The optimality gap is calculated as the objective obtained by RFO minus the objective (or upper bound if solving process is terminated manually) obtained by CPLEX and divide the difference by the objective (or upper bound) obtained by CPLEX. These result display formats are the same for the illustration on the experiments on the grid network and Sioux Fall network later.

Since the grouping of links that need maintenance is random for each RFO iteration, the time needed to solve the same test case for each run will be different and the best solution obtained in each run may also be different from each other. To obtain the representative solving time and objective value for each test case that are not solved optimally by CPLEX in 60 seconds, we run RFO to solve each test case for five times, choose the pair of the objective value and solving time that can represent the average performance of RFO, and compare them with the objective and solving time of CPLEX. The objective values and solving
times of five runs of each test case are listed in the appendix. For the test cases of other networks presented later in this report, we use the same approach to obtain the representative objective and solving time, and append their objectives and solving times in the appendix as well. The solving time of RFO and CPLEX for some test cases are the same because CPLEX was able to solve the entire problem in 60 seconds and the randomized fix-and-optimize procedure did not start.

From Table 2 we can see that even for a 20-link radial network with 50% of the links need maintenance, CPLEX is not able to solve some of the test cases in tolerable amount of time. Also, the RFO heuristic is able to obtain optimal or near-optimal solutions within little amount of time compared to CPLEX. Notice that for the test case when $T = 19$, the objective value from RFO is better than the best feasible solution obtained by CPLEX. To obtain the best feasible solution of this test case, RFO takes less than 7 minutes and the solution dominates the best feasible solution from CPLEX after nearly 3 hours of computation.

A larger network tested is a grid network with 16 nodes, 48 links and 24 OD pairs (network is shown in Figure 6). Grid transportation network structure is frequently found in large modern cities like Phoenix and Vancouver, and their central business districts. Like the radial network tested, the grid network tested also has 50% of links randomly selected as the links to be repaired and the total number of work zones to be scheduled is 52. Details of this grid network (i.e., number of lanes of each link, number of days required to repair a lane, lane capacity, unit flow cost, and whether maintenance is required) and the OD demand are listed in the appendix. The time limits set for solving the entire problem initially and for the FO subproblems are both 60 seconds. RFO is used to solve each test case for five times and representative result is chosen to compare with CPLEX as well. The comparison between the average performance of RFO and the performance of CPLEX is displayed in Table 3 on the next page.
Table 3 below shows that RFO is much more efficient than CPLEX on solving the test cases of the grid network, especially when the test case is difficult to solve. And the solution quality of RFO is also quite good. Usually the percentage of links that need maintenance in a network won’t be as much as 50%. The reason we set the percentage of links to repair 50% for the radial network and grid network tested is because we would like to show how difficult the WZS-TS problem can be and how efficient the RFO is compared to solving the test cases solely by CPLEX.

**TABLE 3: RFO VS CPLEX in Grid Network**

<table>
<thead>
<tr>
<th>Completion Date (T)</th>
<th>Solving Time</th>
<th>Objective Value</th>
<th>Optimality Gap</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RFO</td>
<td>CPLEX</td>
<td>RFO</td>
</tr>
<tr>
<td>12</td>
<td>52.21 sec</td>
<td>52.21 sec</td>
<td>255576</td>
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<tr>
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<td>1.24 min</td>
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</tr>
<tr>
<td>14</td>
<td>5.33 min</td>
<td>25.78 min</td>
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<td>3 min</td>
<td>37.03 min</td>
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<td>17</td>
<td>7.92 min</td>
<td>&gt;14.28 hr.</td>
<td>51773.7</td>
</tr>
</tbody>
</table>

**FIGURE 6: Grid Network.**

![Grid Network Diagram](image.png)
We also test the randomized fix-and-optimize heuristic on the Sioux Falls network which is a real network with 24 nodes, 76 links and 87 OD pairs. There are two sets of problem instances created for the Sioux Falls network, the first set of test cases are based on the scenario that 10% of the links are randomly selected as the links that need maintenance which results in a total number of 16 work zones need to be scheduled. The percentage of links to repair in the second set of test cases is 20% and the total number of work zones to be scheduled is 25. Details of the Sioux-Falls network (i.e., number of lanes of each link, number of days required to repair a lane, lane capacity, unit flow cost, and whether maintenance is required) and the OD demand are listed in the appendix. The time limits on solving the entire problem initially and on solving each FO subproblem are both 40 seconds for first set of test cases, and both are 120 seconds for the second set of test cases. Table 4 and Table 5 give the performance comparison between RFO and CPLEX on the first and second set of test cases respectively. Again, RFO solves each test case five times and the data shown in the table is the one we think represents the average performance of RFO.
From Table 4 we see that when the completion date is small the RFO takes more time to give the final solution than CPLEX does. This is because the problem instance of Sioux Falls network with 10% of links to repair is relatively easy to solve especially when the completion date is small, since the number of integer variables are not large. As the completion date gets larger, the problem instance has more integer variables and gets harder to solve. As a result, the solving times of test cases with larger completion dates are much longer for CPLEX. As a comparison, the solving times for RFO on these test cases increase slightly and the objectives obtained are close to the optimal objectives given by CPLEX. Data in Table 5 also shows that RFO generally has better performance compared to CPLEX when 20% of links of the Sioux Falls network need repair.
TABLE 4: RFO VS CPLEX in Sioux Falls Network with 10% of Links to Repair

<table>
<thead>
<tr>
<th>Completion Date (T)</th>
<th>Solving Time</th>
<th>Objective Value</th>
<th>Optimality Gap</th>
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<tr>
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### TABLE 5: RFO VS CPLEX in Sioux Falls Network with 20% of Links to Repair

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Notice that in Table 4 and 5 the objective obtained by RFO for some test cases is better than the optimal objective obtained by CPLEX. For example, in Table 4 for the test case when T = 23, the objective obtained by RFO is 260302, which is less than the optimal objective 260489.83 from CPLEX. This is because the relative MIP gap tolerance is set to 0.5% for the CPLEX and FO subproblems. CPLEX stops solving process as soon as the relative optimality gap (which is calculated as upper bound minus lower bound and then divide the difference by the upper bound) is under 0.5% and uses the best feasible solution obtained as the optimal solution, which is same for FO subproblems. But because of the randomized grouping of links that need repair, it is possible for a FO subproblem start with a branching node that leads to a better upper bound when the 0.5% relative optimality gap is reached, and this node is not selected or reached by CPLEX in the regular branch-and-bound process. So when the 0.5% relative optimality gap is
reached, the upper bound obtained by CPLEX is not as good as the one obtained by RFO. If we reduce the relative MIP gap tolerance to 0.1% or smaller for CPLEX, CPLEX should be able to obtain the same final solution but certainly with much more time spent on the branch-and-bound process.

5. Conclusion

In this project a mixed integer linear programming model is developed to schedule work zones in transportation service networks (WZS-TS). The model coordinates scheduling of work zones with network-wide perspective to achieve minimum total flow cost of all OD demands throughout the entire project period. The WZS-TS problem is very challenging and CPLEX is not able to solve many test cases, even for a small network with 20 links, after hours of computation on a personal computer. The randomized fix-and-optimize heuristic (RFO) is developed to solve the WZS-TS problem efficiently, which can obtain optimal or near-optimal solutions with much less time compared to solving the WZS-TS problem solely with CPLEX. To illustrate the advantage RFO has over CPLEX, the performance comparison between RFO and CPLEX is made on various tests cases created based on three different networks. The authors note that optimal repair and maintenance of other flow carrying service networks, such as power networks, water distribution networks, and communication systems, may benefit from the model and solution method developed in this project.

This project completes the first phase of the research project “Scheduling Work Zones in Multi-modal Networks”, whose ultimate goal is to study the optimal work zone scheduling that has minimum negative impacts on both transport service vehicle flows and commute vehicle flows. The next phase of the research project is to consider the work zone scheduling in networks serving OD flows of personal cars (commuter vehicles). The flow cost of commuter vehicles is nonlinear and depends on the relation between the link capacity and the total volume of the traffic that is already using the link. The problem of scheduling work zones in networks of commuter vehicles (CV) is similar to the WZS-SV problem discussed in this project on the scheduling part. But for the OD flow routing part, the CV problem is different in the way that each OD pair will choose its own route or set of routes to minimize its own flow cost and all the OD pairs will reach a user equilibrium that every OD pair won’t be able to reduce its flow cost by changing its flow routing unilaterally (Wardrop’s first principle). The CV problem is an interesting but challenging mixed integer nonlinear programming problem that cannot be solved by commercial solvers like CPLEX. Study the CV problem is of great value since the majority users of city transportation networks are commuter vehicles. With the results from the first two phases of the research project, the third phase, which studies the work zone scheduling with considerations for both transport service vehicles and commuter vehicles, and the interactions between these two types of vehicle flows, will be carried out.
Reference


### Appendix

#### Radial Network Link Information

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Objective and Time Consumption of Five Runs by RFO for Each Test Case of the Radial Network

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Objective and Time Consumption of Five Runs by RFO for Each Test Case of the Radial Network

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Pitu B. Mirchandani and Dening Peng
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Objective and Time Consumption of Five Runs by RFO for Each Test Case of the Sioux-Falls Network with 20% of Links to Repair

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<th>Completion Date (T)</th>
<th>Run 1</th>
<th>Run 2</th>
<th>Run 3</th>
<th>Run 4</th>
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C++ Code of the Randomized Fix-and-Optimize Heuristic

//RFO of Workzone Scheduling for Transportation Service Networks -- Two Layer Structure with Network Connectivity Check Ensuring the Feasibility of a Schedule
//Dening Peng
//%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

#include <ilcplex/ilocplex.h>
#include <vector>
#include <time.h>
#include <math.h>
#include <iostream>
#include <fstream>
#include <sstream>
#include <cmath>
#include <random>

IlOStart // macro that allows the use of standard libraries

Pitu B. Mirchandani and Dening Peng
除去MIP回调函数

```cpp
ILOMIPINFOCALLBACK1(AbortCriteria, IloNum, TotalCostFO){
    if(getBestObjValue() > TotalCostFO){
        abort(); // abort current fix and optimize subproblem if the lower bound is larger than the current best FO objective
        cout << endl;
        cout << "The optimization of current FO subproblem is aborted." << endl;
        //cin.get();
    }
}
```

//declare functions

```cpp
void InputData();
void Initialization();
```

//declare struct for data

```cpp
struct generalInfo{
    // from text file "general"
    int numNodes; // number of nodes
    int numLinks; // number of links
    int numODs; // number of Origin-Destination (OD) pairs
    int maxnumLanes; // max number of lanes among all links
    float rho; // ratio of unit extra flow cost to regular flow cost
    int T; // required maintenance completion date (RMCD)
    float theta; // percentage of capacity increase once the lane is repaired, 0<theta<0.5
    //int batchMethod; // the method of batching the links for sequential fix and optimize procedure, 0 is random batching, 1 is batching based on pindex
    float numLinpBat; // number of links per batch
};
```

```cpp
struct linkInfo{
    // from MS Excel file worksheet "link"
```
```cpp
int ID;  // link ID
int FN;  // from node
int TN;  // to node
int n;   // # of lanes of the link
int u;   // capacity upper bound of a lane
float c;   // cost incurred by one unit flow on link when the link is regularly open to serve traffic
int p;   // the number of days needed to repair a lane of the link
bool R;   // indicator of whether the lane needs repair at the beginning of the repair project
int obindex;  // obindex is the sequence of different batches of links to have their maintenance start dates optimized
}

struct ODInfo{
    // from MS Excel file worksheet "od"
    int ID;   // Origin-Destination (OD) pair ID
    int ON;   // origin node
    int DN;   // destination node
    int D;    // traffic flow demand of OD pair each day

    //
    // special structs
    // generalInfo general;  // general parameters
    vector<linkInfo> link;  // link information
    vector<ODInfo> OD;      // OD information

    // parameters
    float timeLimitSA,      // time stopping criteria for SA problem
        timeLimitFO,      // time stopping criteria for FO subproblems
        mipGap,          // optimality gap stopping criteria
        cplexLB;         // lower bound on solution
    float cplexUB;  // upper bound on solution
    float TotalCostFO;  // store the objective value of current iteration of the Fix and Optimize problem
};
```
float TotalCost;  //store the objective value of the best feasible solution in FO relaxation iteration
int sumD;  //the summation of all the OD demand. This number is used to construct the constraint that ensures entirely closed links can not serve any flows
float numBatches;  //total number of batches of links to go through fix and optimize procedure
bool aborted;  //flag variable indicating that at least one FO subproblem of current batching has aborted the optimization process because of LB is greater than TotalCost

vector<vector<vector<float>>> y_o;  //variables used for final solution output
vector<vector<float>> z_o;

vector<vector<vector<bool>>> s_o, x_o, v_o, y_o;  //store the schedule with the best objective so far of upper layer scheduling problem
vector<vector<bool>> w_o;

class SA{
public:
    IloEnv env;
    IloModel SA_Opt;  //model for the SA problem
    IloCplex SA_Solver;  //solver for the SA problem
    IloObjective obj;
    IloExpr expr;
    IloInt k, a, t, m, i, j;
    IloNumVar dummy, dummy1;
    IloRangeArray con_Nonpremp, con_StartDate, con_EntireClose, con_RepairClose, con_ODDemand, con_FlowConserve, con_V, con_W, con_Z;

public:
    /********************************Function that builds the model FO and solves the problem instance*************************/
    void BuildSolveSA(){
        IloModel SA_Opt(env);
        IloCplex SA_Solver(SA_Opt);
        IloExpr expr(env);  //expression used while building model
        IloExpr expr1(env);  //expression used while building mode
        IloNumVar dummy(env, 0, 0, ILOFLOAT);
        IloNumVar dummy1(env, 0, 0, ILOFLOAT);

        //Define Variables
        IloArray<IloArray<IloArray<IloNumVar>>> s(env, general.numLinks);
        IloArray<IloArray<IloArray<IloNumVar>>> s(env, general.numLinks);
        IloArray<IloArray<IloArray<IloNumVar>>> s(env, general.numLinks);
        IloArray<IloArray<IloArray<IloNumVar>>> s(env, general.numLinks);

        //variable for the repair start date of each lane of each link
IloArray<IloArray<IloArray<IloNumVar>> > x(env, general.numLinks);
//variable indicating whether the lane of a link is open or closed
IloArray<IloArray<IloArray<IloNumVar>> > v(env, general.numLinks);
//if the mth lane of link (i,j) is repaired before day t, v_{ijmt}=1, otherwise 0
IloArray<IloArray<IloArray<IloNumVar>> > y(env, general.numLinks);
//flow variable used to ensure the schedule generated is feasible

for (i = 0; i < general.numLinks; i++){
    //link index
    s[i] = IloArray<IloArray<IloNumVar>>(env, general.maxnumLanes);
    x[i] = IloArray<IloArray<IloNumVar>>(env, general.maxnumLanes);
    v[i] = IloArray<IloArray<IloNumVar>>(env, general.maxnumLanes);
    for (m = 0; m < link[i].n; m++){
        //lane index
        s[i][m] = IloArray<IloNumVar>(env, general.T);
        x[i][m] = IloArray<IloNumVar>(env, general.T);
        v[i][m] = IloArray<IloNumVar>(env, general.T);
        for (t = 0; t < general.T; t++){
            //date index
            s[i][m][t] = IloNumVar(env, 0, 1, ILOBOOL);
            x[i][m][t] = IloNumVar(env, 0, 1, ILOBOOL);
            v[i][m][t] = IloNumVar(env, 0, 1, ILOBOOL);
        }
    }
}

for (i = 0; i < general.numLinks; i++){
    //link index
    y[i] = IloArray<IloArray<IloNumVar>>(env, general.numODs);  //OD index
    for (k = 0; k < general.numODs; k++){
        y[i][k] = IloArray<IloNumVar>(env, general.T);
    }
}

w = IloArray<IloArray<IloNumVar>>(env, general.numLinks);
//whether all the lanes of link (i,j) is closed on day t, if it is, w_{ijt}=1; otherwise 0
z = IloArray<IloArray<IloNumVar>>(env, general.numLinks);
for (i = 0; i < general.numLinks; i++){
    w[i] = IloArray<IloNumVar>(env, general.T);
    z[i] = IloArray<IloNumVar>(env, general.T);
    for (t = 0; t < general.T; t++){
        w[i][t] = IloNumVar(env, 0, 1, ILOBOOL);
        z[i][t] = IloNumVar(env, 0, IloInfinity, ILOFLOAT);
    }
}

w = IloArray<IloArray<IloNumVar>>(env, general.numLinks);
//whether all the lanes of link (i,j) is closed on day t, if it is, w_{ijt}=1; otherwise 0

//Define Constraints
//Constraints ensuring once a lane is closed for repair, it won't open to serve the flows until it is fully repaired
IloRangeArray con_Nonpremp(env);
for(i = 0; i < general.numLinks; i++) {
    if(link[i].R > 0.5){
        for(m = 0; m < link[i].n; m++){
for(t = 0; t < general.T; t++){
    expr = dummy;
    if((t - link[i].p + 1) > 0){
        for(a = t - link[i].p + 1; a < t + 1; a++){
            expr += s[i][m][a];
        }
    } else{
        for(a = 0; a < t + 1; a++){
            expr += s[i][m][a];
        }
    }
    expr = x[i][m][t] - expr;
    con_Nonpremp.add(IloRange(env, 0, expr, 0, 0));
    expr.end();
}
}
SA_Opt.add(con_Nonpremp);

//Constraints ensuring each lane of the links to be repaired has a repair start date, and those do not need repair does not have a repair start date
IloRangeArray con_StartDate(env);
for(i = 0; i < general.numLinks; i++) {
    for(m = 0; m < link[i].n; m++){
        expr = dummy;
        for(t = 0; t < general.T; t++){
            expr += s[i][m][t];
        }
        if(link[i].R > 0.5){
            expr = expr - 1;
        }
        con_StartDate.add(IloRange(env, 0, expr, 0, 0));
        expr.end();
    }
} SA_Opt.add(con_StartDate);

//All links that need maintenance have to be repaired before the end of the time horizon, all links that don't need repair can't close any lanes at any time
IloRangeArray con_RepairClose(env);
for(i = 0; i < general.numLinks; i++) {
    for(m = 0; m < link[i].n; m++){
        expr = dummy;
        for(t = 0; t < general.T; t++){
            expr += x[i][m][t];
        }
        if(link[i].R == 1){
            expr = expr - link[i].p;
        }
        con_RepairClose.add(IloRange(env, 0, expr, 0, 0));
        expr.end();
    }
} SA_Opt.add(con_RepairClose);

//Constraint ensuring entirely closed link won't serve flows
IloRangeArray con_EntireClose(env);
for(i = 0; i < general.numLinks; i++){
if(link[i].R > 0.5){
    for(t = 0; t < general.T; t++){
        expr = dummy;
        m = 0;
        for(k = 0; k < general.numODs; k++){
            expr += y[i][k][t];
            m += OD[k].D;
        }
        expr = (1 - w[i][t]) * m - expr;
        IloRange EntireClose(env, 0, expr, IloInfinity, 0);
        con_EntireClose.add(EntireClose);
        expr.end();
        expr1.end();
    }
}
SA_Opt.add(con_EntireClose);
// definition of w[i][t]
IloRangeArray con_W(env);
for(i = 0; i < general.numLinks; i++){
    if(link[i].R > 0.5){
        for(t = 0; t < general.T; t++){
            expr = dummy;
            expr1 = dummy1;
            for(m = 0; m < link[i].n; m++){
                expr1 += x[i][m][t];
            }
            expr = link[i].n - expr1;
            expr = expr - (1 - w[i][t]);
            con_W.add(IloRange(env, 0, expr, IloInfinity, 0));
            expr.end();
            expr = dummy;
            expr = link[i].n * (1 - w[i][t]);
            expr = expr - (link[i].n - expr1);
            con_W.add(IloRange(env, 0, expr, IloInfinity, 0));
            expr.end();
            expr1.end();
        }
    } else{
        for(t = 0; t < general.T; t++){
            expr = w[i][t];
            con_W.add(IloRange(env, 0, expr, 0, 0));
            expr.end();
        }
    }
}
SA_Opt.add(con_W);
// definition of v[i][m][t]
IloRangeArray con_V(env);
for(i = 0; i < general.numLinks; i++){
    for(m = 0; m < link[i].n; m++){
        if(link[i].R > 0.5){
            for(t = 0; t < link[i].p; t++){
                expr = v[i][m][t];
                con_V.add(IloRange(env, 0, expr, 0, 0));
                expr.end();
            }
        }
    }
}
for(t = link[i].p; t < general.T; t++){
    expr = dummy;
    for(a = 0; a < t - link[i].p + 1; a++){
        expr += s[i][m][a];
    }
    expr = expr - v[i][m][t];
    con_V.add(IloRange(env, 0, expr, 0, 0));
    expr.end();
}
else{
    for(t = 0; t < general.T; t++){
        expr = v[i][m][t];
        con_V.add(IloRange(env, 0, expr, 0, 0));
        expr.end();
    }
}
}
SA_opt.add(con_V);

//OD demand constraint
IloRangeArray con_ODDemand(env);
for(k = 0; k < general.numODs; k++){
    for(t = 0; t < general.T; t++){
        expr = dummy;
        expr1 = dummy;
        for(i = 0; i < general.numLinks; i++){
            if(link[i].FN == OD[k].ON){
                expr += y[i][k][t];
            }
            else if(link[i].TN == OD[k].ON){
                expr1 += y[i][k][t];
            }
        }
        expr = expr - expr1;
        con_ODDemand.add(IloRange(env, OD[k].D, expr, OD[k].D, 0));
        expr.end();
        expr1.end();
        expr = dummy;
        expr1 = dummy;
        for(i = 0; i < general.numLinks; i++){
            if(link[i].TN == OD[k].ON){
                expr += y[i][k][t];
            }
            else if(link[i].FN == OD[k].DN){
                expr1 += y[i][k][t];
            }
        }
        expr = expr - expr1;
        con_ODDemand.add(IloRange(env, OD[k].D, expr, OD[k].D, 0));
        expr.end();
    }
}
SA_opt.add(con_ODDemand);

//Flow Conservation Constraint
IloRangeArray con_FlowConserve(env);
for(j = 1; j < general.numNodes + 1; j++){
    for(k = 0; k < general.numODs; k++){
if(OD[k].ON != j && OD[k].DN != j){
    for(t = 0; t < general.T; t++){
        expr = dummy;
        expr1 = dummy1;
        for(i = 0; i < general.numLinks; i++){
            if(link[i].FN == j){
                //OD flow out
                expr += y[i][k][t];
            }
            else if(link[i].TN == j){
                //OD flow in
                expr1 += y[i][k][t];
            }
        }
        expr = expr - expr1;
        con_FlowConserve.add(IloRange(env, 0, expr, 0, 0));
        expr.end();
        expr1.end();
    }
}
SA_Opt.add(con_FlowConserve);
//definition of z[i][t]
IloRangeArray con_Z(env);
for(i = 0; i < general.numLinks; i++){
    for(t = 0; t < general.T; t++){
        expr = z[i][t];
        IloRange Z(env, 0, expr, IloInfinity, 0);
        con_Z.add(Z);
        expr.end();
        expr = dummy;
        expr1 = dummy1;
        for(k = 0; k < general.numODs; k++){
            expr += y[i][k][t];
        }
        for(m = 0; m < link[i].n; m++){
            expr1 += (-x[i][m][t] + general.theta*v[i][m][t]);
        }
        expr1 = (link[i].n + expr1) * link[i].u;
        expr = expr - expr1;
        expr = z[i][t] - expr;
        IloRange Z1(env, 0, expr, IloInfinity, 0);
        con_Z.add(Z1);
        expr.end();
        expr1.end();
    }
}
SA_Opt.add(con_Z);
//Define Objective
expr = dummy;
for(i = 0; i < general.numLinks; i++) {
    for(t = 0; t < general.T; t++){
        expr1 = dummy1;
        for(k = 0; k < general.numODs; k++){
            expr1 += y[i][k][t];
        }
    }
}
expr += link[i].c * expr1 + z[i][t] * general.rho * link[i].c;
expr1.end();
}

IloObjective obj(env, expr, IloObjective::Minimize);
SA_Opt.add(obj);
expr.end();
expr1.end();

// Set Sch_Solver parameter
SA_Solver.setParam(IloCplex::TiLim, timeLimitSA); // restrict the solving time for the entire SA problem to be 60 sec
SA_Solver.setParam(IloCplex::EpGap, mipGap);
SA_Solver.setParam(IloCplex::CutUp, cplexUB);
SA_Solver.setParam(IloCplex::CutLo, cplexLB);
SA_Solver.setParam(IloCplex::Threads, 0);
SA_Solver.setParam(IloCplex::FPHeur, 2); // use feasibility pump heuristic
SA_Solver.setParam(IloCplex::MIPEmphasis, 1); // let cplex put more effort on finding a better feasible solution

cout<<"\n";
cout<<"Constraints of the SA problem are constructed\n";

if (SA_Solver.solve()){
    TotalCost = SA_Solver.getObjValue();
    for(i = 0; i < general.numLinks; i++){
        for(t = 0; t < general.T; t++){
            for(m = 0; m < link[i].n; m++){
                if(SA_Solver.getValue(s[i][m][t]) > 0.5){
                    s_o[i][m][t] =
                    SA_Solver.getValue(s[i][m][t]);
                }

                else if(SA_Solver.getValue(s[i][m][t]) < 0.5){
                    s_o[i][m][t] = 0;
                }

                else if(SA_Solver.getValue(x[i][m][t]) > 0.5){
                    x_o[i][m][t] =
                    SA_Solver.getValue(x[i][m][t]);
                }

                else if(SA_Solver.getValue(x[i][m][t]) < 0.5){
                    x_o[i][m][t] = 0;
                }

                else if(SA_Solver.getValue(v[i][m][t]) > 0.5){
                    v_o[i][m][t] =
                    SA_Solver.getValue(v[i][m][t]);
                }

                else if(SA_Solver.getValue(v[i][m][t]) < 0.5){
                    v_o[i][m][t] = 0;
                }

                if(SA_Solver.getValue(w[i][t]) > 0.5){
                    w_o[i][t] = SA_Solver.getValue(w[i][t]);
                }

                else{
                    w_o[i][t] = 0;
                }
            }
        }
    }
}
C++ Code of the Randomized Fix-and-Optimize Heuristic

```cpp
for (t = 0; t < general.T; t++){
    for (k = 0; k < general.numODs; k++){
        y_o[i][k][t] = SA_Solver.getValue(y[i][k][t]);
    }
    z_o[i][t] = SA_Solver.getValue(z[i][t]);
}
}
}
env.end();
else{
    cout << endl;
    cout << "The original SA problem is not solved" << endl;
    cout << endl;
    //cin.get();
    env.end();
}
}
```

///%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
/// the class of FO Relaxation problem for variable v,s,x,w,y,z
///%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
class FO {
public:
    IloEnv env;
    IloModel FO_Opt;    //model for the FO problem
    IloCplex FO_Solver; //solver for the FO problem
    IloObjective obj;
    IloExpr expr;
    IloInt k, a, t, m, i, j;
    IloNumVar dummy, dummy1;
    IloRangeArray con_Nonpremp, con_StartDate, con_EntireClose, con_RepairClose,
    con_ODDemand, con_FlowConserve, con_V, con_W, con_Z, con_Fix;
    //desicion variables
    IloArray<IloArray<IloArray<IloNumVar>>> s, x, v, y;
    IloArray<IloArray<IloNumVar>> w, z;

public:
    /**********************************************************************************/
    FO() { //Function that builds the model
        void Build() {  
            FO_Opt = IloModel(env);
            FO_Solver = IloCplex(FO_Opt);
            IloExpr expr(env);    //expression used while building model
            IloExpr expr1(env);  //expression used while building model
            IloNumVar dummy(env, 0, 0, ILOFLOAT);
            IloNumVar dummy1(env, 0, 0, ILOFLOAT);

            //Define Variables
            s = IloArray<IloArray<IloArray<IloNumVar>>> (> (env, general.numLinks));
            //variable for the repair start date of each lane of each link
            x = IloArray<IloArray<IloArray<IloNumVar>>> (> (env, general.numLinks));
            //variable indicating whether the lane of a link is open or closed
```
\[ v = IloArray(IloArray(IloArray(IloNumVar) >) >)(env, general.numLinks); \]

//if the mth lane of link (i,j) is repaired before day t, \( v_{ijmt}=1 \), otherwise 0

\[ y = IloArray(IloArray(IloArray(IloNumVar) >) >)(env, general.numLinks); \]

//flow variable used to ensure the schedule generated is feasible

for (i = 0; i < general.numLinks; i++){
    //link index
    s[i] = IloArray(IloArray(IloNumVar) >)(env, general.maxnumLanes);
    x[i] = IloArray(IloArray(IloNumVar) >)(env, general.maxnumLanes);
    v[i] = IloArray(IloArray(IloNumVar) >)(env, general.maxnumLanes);
    for (m = 0; m < link[i].n; m++){
        //lane index
        s[i][m] = IloArray(IloNumVar)(env, general.T);
        x[i][m] = IloArray(IloNumVar)(env, general.T);
        v[i][m] = IloArray(IloNumVar)(env, general.T);
        for(t = 0; t < general.T; t++){ //date index
            s[i][m][t] = IloNumVar(env, 0, 1, ILOBOOL);
            x[i][m][t] = IloNumVar(env, 0, 1, ILOBOOL);
            v[i][m][t] = IloNumVar(env, 0, 1, ILOBOOL);
        }
    }
}

for (i = 0; i < general.numLinks; i++){
    //link index
    y[i] = IloArray(IloArray(IloNumVar) >)(env, general.numODs); //OD index
    for(k = 0; k < general.numODs; k++){
        y[i][k] = IloArray(IloNumVar)(env, general.T);
        for(t = 0; t < general.T; t++){ //date index
            y[i][k][t] = IloNumVar(env, 0, IloInfinity, ILOFLOAT);
        }
    }
}

w = IloArray(IloArray(IloNumVar) >)(env, general.numLinks); //whether all the lanes of link (i,j) is closed on day t, if it is, \( w_{ijt}=1 \); otherwise 0

z = IloArray(IloArray(IloNumVar) >)(env, general.numLinks);
for (i = 0; i < general.numLinks; i++) {
    w[i] = IloArray(IloNumVar)(env, general.T);
    z[i] = IloArray(IloNumVar)(env, general.T);
    for (t = 0; t < general.T; t++){
        w[i][t] = IloNumVar(env, 0, 1, ILOBOOL);
        z[i][t] = IloNumVar(env, 0, IloInfinity, ILOFLOAT);
    }
}

//Define Constraints
//Constraints ensuring once a lane is closed for repair, it won't open to serve the flows until it is fully repaired
IloRangeArray con_Nonpremp(env);
for(i = 0; i < general.numLinks; i++) { //link index
    if(link[i].R > 0.5){
        for(m = 0; m < link[i].n; m++){
            for(t = 0; t < general.T; t++){
            //lane index
                con_Nonpremp(env, s[i][m][t], x[i][m][t], v[i][m][t], y[i][k][t]);
            }
        }
    }
}
expr = dummy;
if((t - link[i].p + 1) > 0){
    for(a = t - link[i].p + 1; a < t + 1; a++){
        expr += s[i][m][a];
    }
} else {
    for(a = 0; a < t + 1; a++){
        expr += s[i][m][a];
    }
}
expr = x[i][m][t] - expr;
con_Nonpremp.add(IloRange(env, 0, expr, 0, 0));
expr.end();
'}

FO_Opt.add(con_Nonpremp);
//Constraints ensuring each lane of the links to be repaired has a repair start date, and
//those do not need repair does not have a repair start date
IloRangeArray con_StartDate(env);
for(i = 0; i < general.numLinks; i++) {
    for(m = 0; m < link[i].n; m++){
        expr = dummy;
        for(t = 0; t < general.T; t++){
            expr += s[i][m][t];
        }
        if(link[i].R > 0.5){
            expr = expr - 1;
        }
        con_StartDate.add(IloRange(env, 0, expr, 0, 0));
        expr.end();
    }
}
FO_Opt.add(con_StartDate);
//All links that need maintenance have to be repaired before the end of the time horizon,
// all links that don't need repair can't close any lanes at any time
IloRangeArray con_RepairClose(env);
for(i = 0; i < general.numLinks; i++) {
    for(m = 0; m < link[i].n; m++){
        expr = dummy;
        for(t = 0; t < general.T; t++){
            expr += x[i][m][t];
        }
        if(link[i].R == 1){
            expr = expr - link[i].p;
        }
        con_RepairClose.add(IloRange(env, 0, expr, 0, 0));
        expr.end();
    }
}
FO_Opt.add(con_RepairClose);
//Constraint ensuring entirely closed link won't serve flows
IloRangeArray con_EntireClose(env);
for(i = 0; i < general.numLinks; i++){
    if(link[i].R > 0.5){

for(t = 0; t < general.T; t++){  
  expr = dummy;  
  m = 0;  
  for(k = 0; k < general.numODs; k++){  
    expr += y[i][k][t];  
    m += OD[k].D;  
  }  
  expr = (1 - w[i][t]) * m - expr;  
  IloRange EntireClose(env, 0, expr, IloInfinity, 0);  
  con_EntireClose.add(EntireClose);  
  expr.end();  
  expr1.end();
}
}
FO_Opt.add(con_EntireClose);

// definition of w[i][t]
IloRangeArray con_W(env);  
for(i = 0; i < general.numLinks; i++){  
  if(link[i].R > 0.5){  
    for(t = 0; t < general.T; t++){  
      expr = dummy;  
      expr1 = dummy1;  
      for(m = 0; m < link[i].n; m++){  
        expr1 += x[i][m][t];  
      }  
      expr = link[i].n - expr1;  
      expr = expr - (1 - w[i][t]);  
      con_W.add(IloRange(env, 0, expr, IloInfinity, 0));  
      expr.end();  
      expr = dummy;  
      expr = link[i].n * (1 - w[i][t]);  
      expr = expr - (link[i].n - expr1);  
      con_W.add(IloRange(env, 0, expr, IloInfinity, 0));  
      expr.end();  
      expr1.end();  
    }  
  }  
  else{  
    for(t = 0; t < general.T; t++){  
      expr = w[i][t];  
      con_W.add(IloRange(env, 0, expr, 0, 0));  
      expr.end();  
    }  
  }
}
FO_Opt.add(con_W);

// definition of v[i][m][t]
IloRangeArray con_V(env);  
for(i = 0; i < general.numLinks; i++){  
  for(m = 0; m < link[i].n; m++){  
    if(link[i].R > 0.5){  
      for(t = 0; t < link[i].p; t++){  
        expr = v[i][m][t];  
        con_V.add(IloRange(env, 0, expr, 0, 0));  
        expr.end();  
      }  
    }  
  }
}

```cpp
expr = dummy;
for (a = 0; a < t - link[i].p + 1; a++){
    expr += s[i][m][a];
}
expr = expr - v[i][m][t];
con_V.add(IloRange(env, 0, expr, 0, 0));
expr.end();
}
else{
    for (t = 0; t < general.T; t++){
        expr = v[i][m][t];
        con_V.add(IloRange(env, 0, expr, 0, 0));
        expr.end();
    }
}
}
FO_Opt.add(con_V);
//OD demand constraint
IloRangeArray con_ODDemand(env);
for (k = 0; k < general.numODs; k++){
    for (t = 0; t < general.T; t++){
        expr = dummy;
        expr1 = dummy;
        for (i = 0; i < general.numLinks; i++){
            if (link[i].FN == OD[k].ON){
                expr += y[i][k][t];
            }
            else if (link[i].TN == OD[k].ON){
                expr1 += y[i][k][t];
            }
        }
        expr = expr - expr1;
        con_ODDemand.add(IloRange(env, OD[k].D, expr, OD[k].D, 0));
        expr.end();
        expr1.end();
        expr = dummy;
        expr1 = dummy;
        for (i = 0; i < general.numLinks; i++){
            if (link[i].TN == OD[k].DN){
                expr += y[i][k][t];
            }
            else if (link[i].FN == OD[k].DN){
                expr1 += y[i][k][t];
            }
        }
        expr = expr - expr1;
        con_ODDemand.add(IloRange(env, OD[k].D, expr, OD[k].D, 0));
        expr.end();
    }
}
FO_Opt.add(con_ODDemand);
//Flow Conservation Constraint
IloRangeArray con_FlowConserve(env);
for (j = 1; j < general.numNodes + 1; j++){
    for (k = 0; k < general.numODs; k++){
        if (OD[k].ON != j && OD[k].DN != j){
            expr = dummy;
            for (i = 0; i < general.numLinks; i++){
                if (link[i].FN == OD[k].ON){
                    expr += y[i][k][t];
                }
                else if (link[i].TN == OD[k].DN){
                    expr1 += y[i][k][t];
                }
            }
            expr = expr - expr1;
            con_FlowConserve.add(IloRange(env, OD[k].D, expr, OD[k].D, 0));
            expr.end();
        }
    }
}
for(t = 0; t < general.T; t++){
    expr = dummy;
    expr1 = dummy1;
    for(i = 0; i < general.numLinks; i++){
        if(link[i].FN == j){
            //OD flow out
            expr += y[i][k][t];
        }
        else if(link[i].TN == j){
            //OD flow in
            expr1 += y[i][k][t];
        }
    }
    expr = expr - expr1;
    con_FlowConserve.add(IloRange(env, 0, expr, 0, 0));
}
}
FO_Opt.add(con_FlowConserve);

//definition of z[i][t]
IloRangeArray con_Z(env);
for(i = 0; i < general.numLinks; i++){
    for(t = 0; t < general.T; t++){
        expr = z[i][t];
        IloRange Z(env, 0, expr, IloInfinity, 0);
        con_Z.add(Z);
        expr.end();
        expr1 = dummy;
        for(k = 0; k < general.numODs; k++){
            expr += y[i][k][t];
        }
        for(m = 0; m < link[i].n; m++){
            expr1 += (-x[i][m][t] + general.theta*v[i][m][t]);
        }
        expr1 = (link[i].n + expr1) * link[i].u;
        expr = expr - expr1;
        expr = z[i][t] - expr;
        IloRange Z1(env, 0, expr, IloInfinity, 0);
        con_Z.add(Z1);
        expr.end();
        expr1.end();
    }
}
FO_Opt.add(con_Z);

//Define Objective
expr = dummy;
for(i = 0; i < general.numLinks; i++){
    for(t = 0; t < general.T; t++){
        expr1 = dummy1;
        for(k = 0; k < general.numODs; k++){
            expr1 += y[i][k][t];
        }
        expr += link[i].c * expr1 + z[i][t] * general.rho * link[i].c;
    }
}
```
expr1.end();
}
IloObjective obj(env, expr, IloObjective::Minimize);
FO_Opt.add(obj);
expr.end();
expr1.end();
//Set Sch_Solver parameter
FO_Solver.setParam(IloCplex::TiLim, timeLimitFO); //restrict
the solving time for each FO subproblem to be 30 sec
FO_Solver.setParam(IloCplex::EpGap, mipGap);
FO_Solver.setParam(IloCplex::CutUp, cplexUB);
FO_Solver.setParam(IloCplex::CutLo, cplexLB);
FO_Solver.setParam(IloCplex::Threads, 0);
FO_Solver.setParam(IloCplex::FPHeur, 2); //use feasibility pump heuristic
to find a good feasible point to start with
FO_Solver.setParam(IloCplex::MIPEmphasis, 1); //let cplex put more effort
on finding a better feasible solution
cout<<"\n";
cout<<"Constraints of the FO problem are constructed\n";
}
/*********************function that adds the constraint fixing the values of
variables of links not in the current FO batch **************/
void Fix(int FOBatchNum){
    con_Fix = IloRangeArray(env);
    for(i = 0; i < general.numLinks; i++){
        if(link[i].obindex != FOBatchNum){
            for(t = 0; t < general.T; t++){
                /*IloRange fixw(env, w_o[i][t], w[i][t], w_o[i][t], 0);
                con_Fix.add(fixw);
                for(m = 0; m < link[i].n; m++){
                    IloRange fixs(env, s_o[i][m][t], s[i][m][t], 0);
                    IloRange fixx(env, x_o[i][m][t], x[i][m][t], 0);
                    IloRange fixv(env, v_o[i][m][t], v[i][m][t], 0);
                    con_Fix.add(fixs);
                    con_Fix.add(fixx);
                    con_Fix.add(fixv);
                }*/
                for(m = 0; m < link[i].n; m++){
                    if(s_o[i][m][t] == 1){
                        IloRange fixs(env, 1, s[i][m][t], 1, 0);
                        con_Fix.add(fixs);
                        con_Fix.add(fixv);
                    }
                }
            }
        }
    }
    FO_Opt.add(con_Fix);
    //cin.get();
}
```
/** function that deletes the constraint fixing the value of scheduling variables of links not in the current FO batch ***/
void Unfix()
{
    FO_Opt.remove(con_Fix);
    con_Fix.end();
}

/** function that solves the Fix and Optimize problem for the optimization batch ***/
bool Solve(int FOBatchNum)
{
    IloCplex::Callback abortcriteria = FO_Solver.use(AbortCriteria(env, TotalCostFO));
    if (FO_Solver.solve())
    {
        cout << endl;
        cout << "The best lower bound of current FO subproblem is: " << FO_Solver.getBestObjValue() << endl;
        //cin.get();
        if(FO_Solver.getBestObjValue() > TotalCostFO)
        {
            aborted = 1;
        }
        if(TotalCostFO > FO_Solver.getObjValue())
        {
            TotalCostFO = FO_Solver.getObjValue();
            for(i = 0; i < general.numLinks; i++)
            {
                if(link[i].obindex == FOBatchNum)
                {
                    for(t = 0; t < general.T; t++)
                    {
                        for(m = 0; m < link[i].n; m++)
                        {
                            if(FO_Solver.getValue(s[i][m][t]) > 0.5)
                            {
                                s_o[i][m][t] = FO_Solver.getValue(s[i][m][t]);
                            }
                            else
                            {
                                s_o[i][m][t] = 0;
                            }
                            if(FO_Solver.getValue(x[i][m][t]) > 0.5)
                            {
                                x_o[i][m][t] = FO_Solver.getValue(x[i][m][t]);
                            }
                            else
                            {
                                x_o[i][m][t] = 0;
                            }
                            if(FO_Solver.getValue(v[i][m][t]) > 0.5)
                            {
                                v_o[i][m][t] = FO_Solver.getValue(v[i][m][t]);
                            }
                            else
                            {
                                v_o[i][m][t] = 0;
                            }
                            if(FO_Solver.getValue(w[i][t]) > 0.5)
                            {
                                w_o[i][t] = FO_Solver.getValue(w[i][t]);
                            }
                            else
                            {
                                w_o[i][t] = 0;
                            }
                        }
                    }
                }
            }
        }
    }
}
```cpp
w_o[i][t] = 0;
}
}
for(i = 0; i < general.numLinks; i++){
    for(t = 0; t < general.T; t++){
        for(k = 0; k < general.numODs; k++){
            y_o[i][k][t] = FO_Solver.getValue(y[i][k][t]);
        }
    }
    z_o[i][t] = FO_Solver.getValue(z[i][t]);
}
//cout << endl;
//cout << "Best schedule for the links in FO batch is
updated" << endl;
//cin.get();
}
cout << endl;
cout << "The Fix and Optimize problem is solved, the objective is
" << TotalCostFO << endl;
stringstream ss8;
stringstream ss9;
stringstream ss10;
string fileName8;
string fileName9;
string fileName10;
ofstream ofileSol8;
ofstream ofileSol9;
ofstream ofileSol10;
ss8 << "FO_s_x.csv";
fileName8 = ss8.str();
ofileSol8.open(fileName8.c_str());
ss9 << "FO_y.csv";
fileName9 = ss9.str();
ofileSol9.open(fileName9.c_str());
ss10 << "FO_z_w_mu.csv";
fileName10 = ss10.str();
ofileSol10.open(fileName10.c_str());
ofileSol8 << "Schedule when time horizon = " << general.T << endl;
ofileSol8 << "OBJECTIVE: " << TotalCostFO << endl;
ofileSol8 << "Values of the s and x matrice: " << endl;
ofileSol8 << "Schedule when time horizon = " << general.T << endl;
ofileSol8 << "OBJECTIVE: " << TotalCostFO << endl;
```

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ofileSol9 << "Values of the y matrix: " << endl;  
for(i = 0; i < general.numLinks; i++){  
  for(k = 0; k < general.numODs; k++){  
    for(t = 0; t < general.T; t++){  
      ofileSol9 << link[i].ID << "," << link[i].FN << "," << link[i].TN << "," << OD[k].ID << "," << OD[k].ON << "," << OD[k].DN << "," << t + 1 << "," << y_o[i][k][t] << endl;  
    }
  }
}
ofileSol9.close();  
ofileSol10 << "Schedule when time horizon = " << general.T << endl;  
ofileSol10 << "OBJECTIVE: " << TotalCostFO << endl;  
ofileSol10 << "Values of the z and w matrices: " << endl;  
ofileSol10 << "Link ID," << "From Node," << "To Node," << "Date," << "z_value," << "w_value" << endl;  
for(i = 0; i < general.numLinks; i++){  
  for(t = 0; t < general.T; t++){  
    ofileSol10 << link[i].ID << "," << link[i].FN << "," << link[i].TN << "," << t + 1 << "," << z_o[i][t] << "," << w_o[i][t] << endl;  
  }
}
ofileSol10.close();  
abortcriteria.end();  
return true;
else{
  cout<<endl;
  cout << "The FO relaxation problem is not solved" << endl;
  cout << endl;
  cin.get();
  return false;
}

/******function that set the solver spend more effort on finding better lower bounds******/
void BLB(){  
  FO_Solver.setParam(IloCplex::MIPEmphasis, 2);
}

/******function that set the solver spend more effort on finding better feasible solutions******/
void BFS(){  
  FO_Solver.setParam(IloCplex::MIPEmphasis, 1);
}

//%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
//main function
//=======================================================================================
==========
int main(int argc, char** argv) {
  time_t startTime, endTime, startTimeC, endTimeC;
double runTime;
int i, j, k, m, t;
int FOBatchNum;
int iter_num; //record the number of iterations performed

InputData(); //obtain network information

InputData();

 Initialization();

 Construct the models of the entire SA problem

startTime = time(0); //time stamp of the starting point of the whole solving process
SA modelSA;
startTimeC = time(0); //time stamp of cplex starts solving the entire SA problem
modelSA.BuildSolveSA();
endTimeC = time(0); //time stamp of cplex stops solving the entire SA problem

modelFO; modelFO.Build();

if(diffTime(endTimeC, startTimeC)<timeLimitSA){
    cout << endl;
    //cin.get();
    goto stop;
}

else{
    cout << endl;
    cout << "Optimality is not reached in " << timeLimitSA << " seconds. Continue to fix and optimize procedure." << endl;
    //cin.get();
    goto FOSTart;
}

FO Iteration

FOStart:
iter_num = 1;
numBatches = 2.0; //start with 2 batches
int numLinpBat;
bool LonSolTime; //flag variable indicating that at least one FO subproblem of current batching has solving time longer than 30 sec

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cout << "Start of FO iteration."
//cin.get();
while(iter_num < 10000){
   vector<int> LintoRep; //the set of links that need repair and
   haven't been assigned to a batch
   for(i = 0; i < general.numLinks; i++){
      if(link[i].R == 1){
         LintoRep.push_back(i); //construct the set with
         the IDs of links that need repair
      }
   }
   numLinpBat = floor(LintoRep.size()/numBatches+0.5); //calculate
   the number of links in a batch
   cout << endl;
   cout << "Number of links per batch: " << numLinpBat << endl;
   //cin.get();
   cout << endl;
   cout << "Total number of links that need repair is " << LintoRep.size() <<
   endl;
   for(j = 1; j < numBatches+1; j++){
      for(k = 1; k < numLinpBat+1; k++){
         int index;
         int ID;
         if(k == 1 && LintoRep.size() == 1){
            ID = LintoRep.at(0); //if there is one
            link[ID].obindex = j - 1;
            LintoRep.erase(LintoRep.begin()); //when the only link
            left is put in the batch in last iteration, the total number of batches will be 1 less
         }
         else{
            srand( time(0) );
            index = rand() % LintoRep.size(); //randomly pick a
            link to be batched in a new batch, put the link in the batch formed in last iteration
            ID = LintoRep.at(index);
            LintoRep.erase(LintoRep.begin());
            link[ID].obindex = j;
         }
      }
   }
endofbatchingFO:
   cout << endl;
   cout << "Number of batches to go through the fix and optimize process: " <<
   numBatches << endl;
   cout << endl;
   //cin.get();
   cout << "All links that need repair are grouped into batches." << endl;
   cout << endl;
   for(j = 1; j < numBatches+1; j++){
      cout << "Batch " << j << ": " << endl;
      for(i = 0; i < general.numLinks; i++){
         if(link[i].obindex == j){
            cout << "Link[" << i+1 << "]" << endl;
         }
      }
   }
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C++ Code of the Randomized Fix-and-Optimize Heuristic

```cpp
LonSolTime = 0; // before the solve the FO subproblems of current batch set the LonSolTime 0
aborted = 0;
for (FOBatchNum = 1; FOBatchNum < numBatches+1; FOBatchNum++){
    modelFO.Fix(FOBatchNum);
    cout << endl;
    cout << "Schedules of links other than the Fix and Optimize Batch " << FOBatchNum << " are fixed." << endl;
    startTimeC = time(0); // record the start time of the solving process of current FO subproblem
    modelFO.Solve(FOBatchNum);
    endTimeC = time(0); // record the end time of the solving process of current FO subproblem
    // cout << endl;
    // cout << difftime(endTimeC, startTimeC) << endl;
    // cin.get();
    if(difftime(endTimeC, startTimeC)>timeLimitFO-1){
        LonSolTime = 1; // if one of the subproblems has solving time longer than 30 sec, set LonSolTime 1
        // cin.get();
    }
    modelFO.Unfix();
    cout << endl;
    cout << "Schedules of links other than the Fix and Optimize Batch " << FOBatchNum << " are unfixed." << endl;
}
cout << endl;
cout << "iter_num = " << iter_num << endl;
cout << "LonSolTime = " << LonSolTime << endl;
cout << "TotalCostFO = " << TotalCostFO << endl;
cout << "TotalCost = " << TotalCost << endl;
cout << "aborted = " << aborted << endl;
// cin.get();
if(((TotalCost-TotalCostFO)/TotalCost > 0.000001){
    TotalCost = TotalCostFO; // if the best objective of current batching is better than the best objective in record, re-batch the links again and do the fix and optimize iteration
    iter_num = iter_num + 1;
}
else{
    if(iter_num < numBatches+1){
        iter_num = iter_num + 1; // if no better schedule if found in current batching and the number of iterations performed is less than 3, random batch again with the same batch number and resolve the FO subproblems
    }
    else{
        if(LonSolTime == 1){
            if(numLinpBat > 3){
                numBatches = numBatches + 1; // increase the number of batches by 1 only if the number of links in per batch is greater than 2
                iter_num = 1; // if no better schedule is found and the solving time of at least one FO subproblem is longer than 30 sec, reset the iter_num
            }
        }
    }
}
```
else{
        goto stop;  //if the
        numLinpBat=3, and no better schedule is found, stop
    }
}

if(LonSolTime==0){
    modelFO.BLB();   //if no FO subproblem takes long solving time in
    current batching, set the solver on finding better lower bounds
}
else if(LonSolTime==1){
    modelFO.BFS();   //if there is FO subproblem taking long solving
    time in current batching, set the solver on finding better feasible solutions
}

stop:
stringstream ss1;
stringstream ss2;
stringstream ss3;
string fileName1;
string fileName2;
string fileName3;
ofstream ofileSol1;
ofstream ofileSol2;
ofstream ofileSol3;
ss1 << "c_network_repair_s_x.csv";
fileName1 = ss1.str();
ofileSol1.open(fileName1.c_str());
ss2 << "c_network_repair_y.csv";
fileName2 = ss2.str();
ofileSol2.open(fileName2.c_str());
ss3 << "c_network_repair_z_w.csv";
fileName3 = ss3.str();
ofileSol3.open(fileName3.c_str());

for(i = 0; i < general.numLinks; i++){
    for(m = 0; m < link[i].n; m++){
        for(t = 0; t < general.T; t++){
            ofileSol1 << link[i].ID << "\n" << link[i].FN << "\n" << link[i].TN << "\n" << m + 1 << "\n" << t + 1 << "\n" << s_o[i][m][t] << "\n" << x_o[i][m][t] << "\n" << v_o[i][m][t] << endl;
        }
    }
    ofileSol1.close();
}
```cpp
ofileSol1.close();
ofileSol2 << "Schedule when time horizon = " << general.T << endl;
ofileSol2 << "OBJECTIVE: " << TotalCost << endl;
ofileSol2 << "Values of the y matrix: " << endl;
ofileSol2 << "Link ID," << "From Node," << "To Node," << "OD ID," << "Origin
Node," << "Destination Node," << "Date," << "y_value," << endl;
for(i = 0; i < general.numLinks; i++){
    for(k = 0; k < general.numODs; k++){
        for(t = 0; t < general.T; t++){
            ofileSol2 << link[i].ID << "," << link[i].FN << "," << link[i].TN << "," << OD[k].ID << "," << OD[k].ON << "," << OD[k].DN << "," << t + 1 << "," << y_o[i][k][t] << endl;
        }
    }
    ofileSol2.close();
}
ofileSol3 << "Schedule when time horizon = " << general.T << endl;
ofileSol3 << "OBJECTIVE: " << TotalCost << endl;
ofileSol3 << "Values of the z and w matrice: " << endl;
ofileSol3 << "Link ID," << "From Node," << "To Node," << "Date," << "z_value," << "w_value" << endl;
for(i = 0; i < general.numLinks; i++){
    for(t = 0; t < general.T; t++){
        ofileSol3 << link[i].ID << "," << link[i].FN << "," << link[i].TN << "," << t + 1 << "," << z_o[i][t] << "," << w_o[i][t] << endl;
    }
}ofileSol3.close();
endTime = time(0); //time stamp of the time point when the entire program
ends
runTime = difftime(endTime, startTime) / 60;
cout << endl;
cout << "Total Solving Time: \t" << runTime << " min" << endl;
cin.get();
return 0;
}

//=======================================================================================
 //=InputData function
 //=======================================================================================
 //========================================================================================
void InputData(){
    //if stream
    ifstream modelParams,
        linkData,
        ODData;
    //read general data
    modelParams.open("Input_modelParameters.txt");
    modelParams >> timelimitSA;
    modelParams >> timelimitFO;
```
modelParams >> mipGap;
modelParams >> cplexUB;
modelParams >> cplexLB;
modelParams >> general.numNodes;
modelParams >> general.numLinks;
modelParams >> general.numODs;
modelParams >> general.maxnumLanes;
modelParams >> general.rho;
modelParams >> general.T;
modelParams >> general.theta;
modelParams.close();

// read link data
linkData.open("Input_links.txt");
int j = 1;
int i = 0;
for(i = 0; i < general.numLinks; i++){
    link.push_back(LinkInfo());
    linkData >> link[i].ID;
    linkData >> link[i].FN;
    linkData >> link[i].TN;
    linkData >> link[i].n;
    linkData >> link[i].u;
    linkData >> link[i].c;
    linkData >> link[i].p;
    linkData >> link[i].R;
    if(link[i].R == 1){
        link[i].obindex = 0;
        link[i].obindex = 0;
    }
}
linkData.close();

// read generator parameters
ODData.open("Input_ODs.txt");
int D = 0;
for(int k = 0; k < general.numODs; k++){
    OD.push_back(ODInfo());
    ODData >> OD[k].ID;
    ODData >> OD[k].ON;
    ODData >> OD[k].DN;
    ODData >> OD[k].D;
    D = D + OD[k].D;
}
sumD = D;
ODData.close();

    cout << "Import Data complete." << endl;
}

//=======================================================================================
=============
//InitSolution function
//=======================================================================================
============
void Initialization(){
    int i, j, m, k, t;
    TotalCost = 1e38;
    TotalCostFO = 1e38;
for (i = 0; i < general.numLinks; i++)
{
    // link index
    s_o.push_back(vector<vector<bool>>(0));
    x_o.push_back(vector<vector<bool>>(0));
    y_o.push_back(vector<vector<float>>(0));
    v_o.push_back(vector<vector<bool>>(0));

    for (m = 0; m < link[i].n; m++)
    {
        // lane index
        s_o[i].push_back(vector<bool>());
        x_o[i].push_back(vector<bool>());
        v_o[i].push_back(vector<bool>());

        for (t = 0; t < general.T; t++)
        {
            // date index
            s_o[i][m].push_back(0);
            x_o[i][m].push_back(0);
            v_o[i][m].push_back(0);
        }
    }

    for (k = 0; k < general.numODs; k++)
    {
        // OD index
        y_o[i].push_back(vector<float>(0));

        for (t = 0; t < general.T; t++)
        {
            y_o[i][k].push_back(0.0);
        }
    }
}

for (i = 0; i < general.numLinks; i++)
{
    // link index
    z_o.push_back(vector<float>());
    w_o.push_back(vector<bool>(0));

    for (t = 0; t < general.T; t++)
    {
        z_o[i].push_back(0.0);
    }
}
}

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