Comprehensive Exam – Microeconomic Theory

August 2015

Answer the questions as precisely as you can. But when in doubt explain more rather than less. You will have to explain (not just state) your answers to get full credit.

Question 1.

Consider the following direct utility function:

\[ U = -\frac{\alpha^2}{q_1} - \frac{\beta^2}{q_2} \]

Where \( \alpha \) and \( \beta \) are positive coefficients. Let the expenditure constraint be

\[ m - P*q = 0, \]

where \( m \) is the individual’s income, \( P = (p_1, p_2) \) is the price vector, and \( q = (q_1, q_2) \) is the quantity vector.

a. Derive the Marshallian demand functions for \( q_1 \) and \( q_2 \) using the Lagrange method.

b. Calculate the determinant of the Hessian matrix for part (a) and show that it has the correct sign for constrained utility maximization.

c. Derive the expression for the own-substitution effect for \( q_1 \) and \( q_2 \) using the Slutsky equation.

d. Use the results in part © and demonstrate that the own-substitution effect is negative for \( q_1 \) and \( q_2 \). Explain the economic significance of this finding.
Question 2.

Suppose that the demand for stilts is given by

\[ D(P) = 1500 - 50P \]

and that long-run total operating costs of each stilt-making firm in a competitive industry is given by

\[ C = 0.5q^2 - 10q \]

Entrepreneurial talent for stilt-making is scarce. The supply curve for entrepreneurs is given by

\[ Q_s = 0.25W \]

Where W is the annual wage paid.

Suppose also that each stilt-making firm requires one (and only one) entrepreneur (hence the quantity of entrepreneurs hired is equal to the number of firms). Long-run total costs for each firm are hence given by

\[ C = 0.5q^2 - 10q + W \]

a. What is the long-run equilibrium quantity of stilts produced? How many stilts are produced by each firm? What is the long-run equilibrium price of stilts? How many firms will there be? How many entrepreneurs will be hired, and what is their wage?

b. Suppose that the demand curve for stilts shifts outward to \( D(P) = 2428 - 50P \).

Answer the questions posed in part (a).

c. Sketch your results. Show the approximate shape of the long-run supply curve. Why does the curve have this shape?
**Question 3.**

Consider the following game. Player 1 can pay some amount $X > 0$ or $Y > 0$. If he pays $X$ then he does not observe what 2 does next but he pays $Y$ then he does observed what player 2 does. Player 2 knows how much player 1 has paid before making his choice. The complete extensive form of the game is shown below.

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a) List all the (pure) strategies for each of the players.

b) Assume that $X=3.5$ and $Y=3$. Find all the pure strategy SPNEs of this game. After formally showing the SPNEs suggest an intuitive explanation for your results.

c) Now, let $X$ and $Y$ be any positive real number. Find all the pure-strategy SPNE in the game as a function of $x$ and $y$. That is, for all possible combinations of $x, y > 0$, list all the pure-strategy SPNE.
d) Now consider the game where 2 does not observe what payment (X or Y) 1 has chosen. Draw the extensive form of the game.

e) Find all the pure-strategy SPNEs in the new game as a function of X and Y. That is, for all possible combinations of X, Y > 0, list all the pure-strategy SPNE. Discuss how this set compares to the set of pure-strategy SPNE in the original game.

Question 4.

Two persons are bidding on an object for auction. Each person has independent, private valuations for the object that are drawn from a uniform distribution over the interval [0, 1].

a) Suppose that the auction is sealed-bid, first price. Find the symmetric Bayesian Nash Equilibrium of this game, in which each player uses the same bid function.

b) Now suppose that it is a second price auction [The highest bidder wins the object but he pays the second highest bid, not his own]. Find the Nash Equilibrium in this game. (hint: look for equilibrium by eliminating weakly dominated strategies)

c) Using the equilibrium bid functions, prove that both types of auctions yield the same expected revenue for the seller.