Comprehensive Exam – Microeconomic Theory

August 2014

Answer the questions as precisely as you can. But when in doubt explain more rather than less. You will have to explain (not just state) your answers to get full credit.

Question 1.

Let \( E(P_1, P_2, U) = UP_1P_2/(P_1 + P_2) \) be an expenditure function.

a. Derive the Marshallian demand functions for goods 1 and 2.
b. Derive the Hicksian demand functions for goods 1 and 2.
c. Verify the Slutsky equation holds for the first good involving a change in the price of the second good.
d. Verify the Slutsky equation holds for the second good involving a change in the price of the second good.

Question 2.

Consider the following CES production function:

\[
q = f(x_1, x_2) = 2[0.5x_1^{0.4} + 0.5x_2^{0.4}]^{2.5}
\]

a. Derive the cost function.
b. Verify that the determinant of the Hessian matrix for constraint cost minimization has the correct sign.
c. Calculate the elasticity of substitution.
d. Apply La Chatelier principal to derive the SR and LR factor demand functions for the first input.
3. A monopolist (P1) faces a potential entrant (P2). First P1 chooses a level of investment \( I > 0 \). This decision is observed by P2. Next, P2 decides whether or not to enter the market. If P2 does not enter, P1 remains a monopolist and earns profits equal to \((2I + 1)\pi_m - I\), while P2 earns zero and the game ends. If P2 enters, P1 observes P2’s decision and decides whether or not to exit the market. If it exits, it earns \( \theta - I \), while P2 earns profits equal to \( \pi_m - F \), where \( F \) is the cost of entry. If P1 does not exit, then the market is shared by the two firms with corresponding profits of \((2I + 1)\pi_d - I\) for P1 and \( \pi_d - F \) for P2. Assume throughout that \( \pi_d < F < \pi_m \) and that, if indifferent between exiting and staying, P1 chooses to stay and all this is common knowledge.

a. Show the structure of the game by sketching the extensive form.

b. Find the subgame-perfect Nash equilibrium (SPNE) of the game in the following two cases. In each case calculate also the equilibrium profits of both firms. Explain your answers.

i) \( \pi_d = \frac{1}{18}, \pi_m = \frac{1}{8}, \theta = \frac{1}{16} \)

ii) \( \pi_d = \frac{1}{18}, \pi_m = \frac{1}{8}; \theta = \frac{3}{16} \)

4. Consider the following incomplete information game.

a. What are all the pure-strategy Bayesian Nash Equilibria (BNE) of the game?

b. What are all the Perfect Bayesian Equilibrium (PBE) of the game? If the set of BNE and the set of PBE is different please explain why some of the equilibria is in one set but not in the other.

c. If you refine the set of PBE(s) (i.e. restrictions R5 and R6), does that change the set of PBE(s) you found in part (ii)? Explain.