Macroeconomics Comprehensive Exam  
Winter 2016

Question 1: Static Modeling

1. Consider the consumption function $C = C(Y - T, r)$, where $C$ denotes consumption, $Y$ denotes income, $T$ denotes the level of taxes, and $r$ denotes real interest rate level. Present restrictions on this behavioral equation that are reasonable, and then state in words what this equation (along with your restrictions) is saying about consumption behavior.

2. Suppose $C(Y - T, r) = [Y - T]^\alpha r^{-\beta}$, so $C = [Y - T]^\alpha r^{-\beta}$. Show that $\alpha$ and $\beta$ are elasticities.

3. Perform the calculation needed in order to describe how the growth rate of consumption is related to the growth rates of income and taxes. Comment using words regard what can be learned from the calculation.

4. Add an investment function of your choice to the consumption function $C = C(Y - T, r)$, and also add the market clearing condition $Y = C + I + G$. Classify the variables for the three equations in a Keynesian manner.

5. For three equation Keynesian model you present for the previous question, find the multipliers $dY/dG$ and $dY/dT$.

6. Using words, interpret the two multiplier results you obtain for the previous question. What do they say about how the economy works?

7. Add the equation $G = T$, so a balanced budget is required. Recalculate $dY/dG$ and compare your new result to the previous result for $dY/dG$. Explain, using words, why the two multipliers are different.

8. Reduce the three equation model presented for question 4 to an IS equation. Add the LM equation $M = PL(Y, r)$, where $M$ is the nominal money supply, $P$ is the price level, and $L(Y, r)$ is a real money demand function that shows how the real money demand level depends upon the real income level and real interest rate level. Classify the variables in your two equation model in a Keynesian manner.

9. For this two equation model, find the multiplier $dY/dG$. Compare this multiplier to the result you obtained to answer question 5. In providing the comparison, discuss the crowding out concept.

10. Reclassify the variables in the IS and LM equations in question (8) in a Classical manner.

11. Add a production function and labor market to the classical model you specify in question 10, so that you have a classical labor market. That is, the wage adjusts to ensure labor supply equals labor demand, so the economy experiences full employment. Classify the variables for your extended model.

12. Recalculate $dY/dG$ for this extended classical model. Again, discuss using words crowding out. You may supplement your words with diagrams if you like.
Question 2: Solow-Swan Model with a Bubble

Consider the following growth model, where the variables are defined as in ECON 703. The variable $B$ is the level of “consumption loans” received by households obtained by borrowing from the capital market. As you can see in equation (1), this consumption loan possibility implies households not only can finance consumption and saving with income but also with consumption loans. The capital market equilibrium equation (2) also implies saving not only finances investment but also finances consumption loans. Finally, equation (9) indicates the total amount of interest $rB$ paid on consumption loans is financed by issuing additional consumption loans in the amount $B'$.

(1) $C + S = Y + B$
(2) $S = I + B$
(3) $Y = F(AL, K)$
(4) $F_{AL} > 0, F_{K} > 0, F_{AL, AL} < 0, F_{KK} < 0$
(5) $K' = I - \delta K, \quad 0 < \delta < 1$
(6) $r = F_{K}(AL, K)$
(7) $L' = nL, \quad n > 0$
(8) $A' = gL, \quad g > 0$
(9) $B' = rB$

Endogenous (9): $l, c, s, y, k', r, l', a', b'$
Predetermined (4): $k, l, a, b$
Initial Conditions (4): $k_0, l_0, a_0, b_0$
Exogenous (4): $n, s, \delta, g$

1. Show the constant returns to scale assumption for the production function implies the production function (4) can be transformed into the intensive form production function $y = f(k)$, where $y = Y/AL$ and $k = K/AL$.
2. Remembering that $F_{K}(AL, K) = \partial[F(AL, K)]/\partial K$, use the facts that $f(k) = f(K/AL)$ and $f(K/AL) = F(1, K/AL)$ to show $r = F_{K}(AL, K)$ assumed in equation (6) implies $r = f'(k)$.
3. Let the “per capita” consumption loan level be defined by $b = B/AL$. Use this definition to show equation (9) can be transformed into the intensive form equation $b' = [r - (g + n)]b$.
4. Use combinations of the model equations as you choose to derive the intensive form basic dynamic equation $k' = sY + b - [g + n + \delta]k$.
5. Using $k' = sY + b - [g + n + \delta]k$ from question 4 and $y = f(k)$ from question 1, note that $k' = sf(k) + b - [g + n + \delta]k$. Consider this single equation, thinking of $k'$ as endogenous and $k$ predetermined. Assuming the Inada conditions hold for the function $f(k)$ as in class, show in a diagram that there is a single steady state for the per capita capital stock level $k$.
6. Prove that the single steady state identified in the previous question is stable, if the Inada conditions hold.
7. Using the steady state for the single equation model presented in the last question, find the comparative static multiplier $dk/db$. Use your result to discuss how a change in the per capital consumption loan “bubble” $b$ impacts the steady state per capita capital stock level $k$.
8. Together, equation (1) and the production relationship $y = f(k)$ imply $c = [1 - s]f(k) + b$. Remembering from question 7 that the steady state level $k$ depends upon the level of $b$, differentiate this last condition and find the condition that must hold in order for the steady state consumption level $c$ to be maximized. Then, use your multiplier result $dk/db$ and the condition $r = f'(k)$ to find a condition on the interest rate level $r$ that will hold when the steady state consumption level $c$ is maximized. (This is the golden rule for this model.)
9. Consider the four intensive form equations presented in questions 1-4 as a system, which determines the four endogenous variables $y, r, b'$ and $k'$. Use any remaining time you have to examine the dynamics of the whole system.
Question 3: Asset Pricing, Lucas’ Tree Model-Trees in the Utility function

Consider the following version of the Lucas Tree model. There are two kinds of trees. The first kind is ugly and gives no direct utility in itself, but yields a stream of fruit \( \{d1t\} \), where \( d1t \) is a random process obeying a first order Markov process. The fruit is non-storable and yields utility. The second kind of tree is beautiful and so yields utility in itself. This tree also yields a stream of the same kind of fruit \( \{d2t\} \), where it happens that: \( d2t = d1t = (1/2)d \) for all \( t \), so that the physical yields of the two kinds of trees are equal. There is one of each kind of tree for each of the \( N \) individuals in the economy. Trees last forever, but the fruit is non-storable. Trees are the only source of fruit.

Each of the \( N \) individuals in the economy has preferences described by:

\[
E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, s_{2t})
\]

Where: \( u(c_t, s_{2t}) = \ln(c(t) + \nu \ln(s_{2t})) \); \( \nu \geq 0 \). \( C(t) \) is the consumption of fruit at time \( t \), and \( s_{2t} \) is the stock of beautiful trees owned at the beginning of period \( t \). The owner of a tree of either kind \( i \) at the beginning of period \( t \) receives the fruit produced, \( d_{it} \), during period \( t \). Let \( p_it \) be the price of a tree of type \( i = 1, 2 \). Let \( R_{it} \) be the gross rate of return on trees of type \( i \) held from \( t \) to \( t+1 \). Consider a competitive equilibrium with markets in stocks for each kind of tree.

You must:

a) Provide pricing formulas for \( p_{1t} \) and \( p_{2t} \).

b) Prove that if \( \nu > 0 \), \( R_{1t} > R_{2t} \) for all \( t \). That is, prove that, in equilibrium, beautiful trees are dominated in rate of return by ugly trees. Explain this result with words.

Question 4: Money and growth models plus the IS-LM

a. Add money to the basic Ramsey model. Introduce money as an argument in the Cost of Transactions function or in the utility function (in this latter case, make sure money and consumption enter utility in an additively separable way). Arrive at the Euler Equation (EE) for consumption. How does the EE compare to the one of the standard Ramsey model? Is money super-neutral here? Why or why not?

b. Now introduce money in utility in a non-separable way and assume that money and consumption are complements, utility-wise (the cross partial second derivative of utility is positive). Is money super-neutral here? Why or why not? Explain.

c. Drop capital from the model and assume money enters utility in an additively separable way. Specify the market clearing conditions in the market for: (i) goods, (ii) money; (iii) bonds.

d. Show that the EE for consumption implies the existence of a forward-looking IS-type equation (i.e., a mathematical expression in which real GDP, \( y \), and the real interest rate, \( r \), are inversely related). How is the IS equation implied by the EE different from the standard IS equation presented in intermediate macro textbooks?

e. Show that the first order condition for money demand coupled with the market clearing condition in the market for goods, the market clearing condition in the market for bonds, and the market clearing condition in the market for money jointly imply an LM-type equation (i.e., a
mathematical relation in which money demand depends positively on y, negatively on i, the *nominal* interest rate)

f. Are there any conditions under which the LM equation would give rise to a vertical LM curve here? If so, what are those conditions? How effective would fiscal policy be in this case?

g. Are there any conditions under which the LM equation would give rise to a perfectly flat LM curve here? If so, what are those conditions? How effective would monetary policy be in this case?

h. Set up the baseline endowment economy MIU model with no capital and no production and show analytically why hyper-deflations cannot take place in theory (they certainly do not happen in practice).

i. How many equilibria do monetary models typically have? Why is that? Explain. What requirement is it imposed on the stability of the equilibria? Why is that?

j. Is a very low rate of money supply growth sufficient to rule out the existence of hyperinflations? Why or why not? Prove your point analytically.

k. Which are the critical Inada condition(s) that should not be imposed on the utility of money balances function if hyperinflations are to be allowed to happen? In other words, what is the critical Inada condition that rules out hyperinflations? Why? Explain.