Macroéconomie Comprehensive Exam  
Winter 2015

Question 1: Understanding the Macroeconomy with an IS-LM Model

Consider the economy modeled by the equations below. The real consumption level $C$ depends upon the level of disposable income $Y - T$, where $Y$ is the real income or real GDP level of the economy and $T$ is the real taxation level, and consumption also depends upon the real wealth $X$ held by consumers. The economy’s real investment level $I$ depends upon the real interest rate level $r$. The real money demand level $L$ in the economy depends upon the real interest rate level $r$ and real income level $Y$.

\begin{align*}
(1) \quad & C = C(Y - T, X) \\
(2) \quad & I = I(r) \\
(3) \quad & L = L(r, Y)
\end{align*}

1. Present restrictions on the consumption, investment and money demand functions, and use words to explain why the restrictions you present make economic sense.
2. Present one diagram where you plot consumption as it depends upon disposable income and a second diagram where you plot consumption as it depends upon wealth.
3. Take the total differential of the consumption function. Use your result, the restrictions you presented above for the consumption function, and the two diagrams you presented above to explain how consumption is affected by disposable income and wealth. Specifically, for each of the two diagrams, explain when there is a move along the curve versus when the curve shifts, and relate the diagrams as carefully as you can to your total differential.
4. Add two equilibrium conditions to the system of equations, one for the capital market and one for the money market. Add variables as necessary or as sensible, and use words to define any new variable you add. At minimum, add the real government purchases level $G$, recognizing this is a model with government.
5. Classify the variables of your complete 5 equation model so the model has a Keynesian perspective on how the economy works.
6. Find the multipliers $dY/dX$ and $dY/dG$
7. Compare the two multipliers you calculated in the previous question. Identify which multiplier is bigger. Use words to explain why one multiplier is bigger than the other.
8. Use an IS-LM diagram and either of the multipliers you calculated to explain the crowding out concept. Further, explain why this type of crowding out is not so serious in that it can be prevented if the Federal Reserve gets involved.
9. Reclassify the variables of this model, continuing to keep a Keynesian perspective, after adding the following two equations. Add the production function $Y = f(N)$, with the restrictions $f'(N) < 0$ and $f''(N) < 0$, and add the equation $W = Pf'(N)$ which is the assumption that labor is paid a nominal wage $W$ equal to the marginal revenue product $Pf'(N)$.
10. Find the multipliers $dY/dX$ and $dY/dG$ for the expanded seven equation model.
11. Again, use your multipliers to comment on crowding out. Explain why there are two types of crowding out in these multipliers. Explain why the new type of crowding out is more serious.
12. Use this entire model to explain the Keynesian perspective of how the economy works, and as you have time compare it to the Classical perspective (just using words, no modeling).
Question 2: Solow-Swan Model with a Bubble

Consider the following growth model, where the variables are defined as in ECON 703. The variable $B$ is the level of “consumption loans” received by households obtained by borrowing from the capital market. As you can see in equation (1), this consumption loan possibility implies households not only can finance consumption and saving with income but also with consumption loans. The capital market equilibrium equation (2) also implies saving not only finances investment but also finances consumption loans. Finally, equation (9) indicates the total amount of interest $rB$ paid on consumption loans is financed by issuing additional consumption loans in the amount $B'$.

(1) $C + S = Y + B$
(2) $S = I + B$
(3) $S = sY$. $0 < s < 1$
(4) $Y = F(AL, K)$ $\lambda Y = F(\lambda AL, \lambda K)$, $F_{AL} > 0$, $F_{K} > 0$, $F_{AL, AL} < 0$, $F_{KK} < 0$
(5) $k' = I - \delta K$, $0 < \delta < 1$
(6) $r = F_{b}(AL, K)$
(7) $L' = nL$, $n > 0$
(8) $A' = gL$, $g > 0$
(9) $B' = rB$

Endogenous (9): $I$, $C$, $S$, $Y$, $k'$, $r$, $L'$, $A'$, $B'$

Predetermined (4): $K$, $L$, $A$, $B$

Initial Conditions (4): $K_0$, $L_0$, $A_0$, $B_0$

Exogenous (4): $n$, $s$, $\delta$, $g$

1. Show the constant returns to scale assumption for the production function implies the production function (4) can be transformed into the intensive form production function $y = f(k)$, where $y = Y/AL$ and $k = K/AL$.
2. Remembering that $F_{k}(AL, K) = \partial[F(AL, K)]/\partial K$, use the facts that $f(k) = f(K/AL)$ and $f(K/AL) = F(1, K/AL)$ to show $r = F_{k}(AL, K)$ assumed in equation (6) implies $r = f'(k)$.
3. Let the “per capita” consumption loan level be defined by $b = B/AL$. Use this definition to show equation (9) can be transformed into the intensive form equation $b' = [r - (g + n)]b$.
4. Use combinations of the model equations as you choose to derive the intensive form basic dynamic equation $k' = sy + b - [g + n + \delta]k$.
5. Using $k' = sy + b - [g + n + \delta]k$ from question 4 and $y = f(k)$ from question 1, note that $k' = sf(k) + b - [g + n + \delta]k$. Consider this single equation, thinking of $k'$ as endogenous and $k$ predetermined. Assuming the Inada conditions hold for the function $f(k)$ as in class, show in a diagram that there is a single steady state for the per capita capital stock level $k$.
6. Prove that the single steady state identified in the previous question is stable, if the Inada conditions hold.
7. Using the steady state for the single equation model presented in the last question, find the comparative static multiplier $dk/db$. Use your result to discuss how a change in the per capita consumption loan “bubble” $b$ impacts the steady state per capita capital stock level $k$.
8. Together, equation (1) and the production relationship $y = f(k)$ imply $c = [1 - s]f(k) + b$. Remembering from question 7 that the steady state level $k$ depends upon the level of $b$, differentiate this last condition and find the condition that must hold in order for the steady state consumption level $c$ to be maximized. Then, use your multiplier result $dk/db$ and the condition $r = f'(k)$ to find a condition on the interest rate level $r$ that will hold when the steady state consumption level $c$ is maximized. (This is the golden rule for this model.)
9. Consider the four intensive form equations presented in questions 1-4 as a system, which determines the four endogenous variables $y$, $r$, $b'$ and $k'$. Use any remaining time you have to examine the dynamics of the whole system.
Question 3. Conceptual and analytical inter-relations across basic dynamic macroeconomic models. Part 1: consumption, taxation and asset pricing

1. Basic consumption theory
   a. Set up a basic dynamic endowment economy, representative agent model to illustrate the most basic of all inter-temporal decisions in macroeconomics, namely the tradeoff between present and future consumption. Start with the structural model equations. Hint: remember you need: (i) a functional to maximize (it has to be bounded from above to be meaningful); (ii) an initial condition for the state variable; (iii) an evolution equation describing the temporal trajectory of the state variable, and (iv) a terminal condition, (or transversality condition in infinite horizon).
   b. Set up the optimization problem using the Lagrangian method.
   c. Obtain the IBC (inter-temporal budget constrain) and explain it.
   d. Arrive at the Euler equation and explain it in words.
   e. Specialize the Euler equation for the cases of CRRA utility and CARA utility. Explain both of them in words. CARA utility has at least one strongly counter-actual implication. Which one? Explain.

2. Basic asset pricing theory
   a. Extend the endowment economy model developed in a. above to be able to explain not just consumption-saving decisions, but also portfolio allocation decisions by adding a risky asset to the range of available assets. Setup the optimization problem using Bellman’s equation method and obtain the two relevant first order conditions. Invoke envelope’s theorem, some statistical properties of random variables, and arrive to the fundamental reduced-form solution characteristic of modern asset pricing. Explain it with words. Explain hedging.
   b. Specialize the asset pricing model for the cases of log utility and linear utility. Explain the reduced-form equations with words.

3. Optimal dynamic taxation

Question 4. Conceptual and analytical inter-relations across basic dynamic macroeconomic models. Part 2: Growth and Money models

4. Exogenous growth theory
   a. Add production (via an aggregate technology that satisfies all Inada conditions plus constant returns to scale) to the basic model of point a. above, so as to be able to discuss (exogenous) economic growth theory. Start by setting up the basic Ramsey model in continuous time. Arrive at the system of differential equations characterizing the model and draw the phase diagram. Show analytically how to obtain the level of capital and consumption that maximize steady state utility.
   b. What happens if the planning horizon is finite in the Ramsey model? Why do we need an infinite horizon?
   c. Set up the basic Solow model now. Apply the golden rule and obtain the level of capital that maximizes steady state consumption. Identify the coordinates associated with this (c.k) pair on
the phase diagram you drew for Ramsey’s model. Compare the two optimal points (the modified golden rule and the golden rule)

d. Obtain the golden rule result in Ramsey’s model. Show analytically how you can do that. 
Hints: (1) Show that Solow’s reduced-form solution differential equation can be transformed via a few steps into Ramsey’s per capita resource constraint differential equation; (2) Analyze Ramsey’s pair of differential equations in steady state and find the condition such that steady state consumption and capital are the same as the ones in the golden rule.

e. Which exogenous growth model is better, Ramsey’s or Solow’s? Why? Explain what advantages and disadvantages they have, both in terms of positive and normative economics.

5. Money and growth models

a. Add money to the basic Ramsey model. Introduce money as an argument in the Cost of Transactions function or in the utility function (in this latter case, make sure money and consumption enter utility in an additively separable way). Arrive at the Euler Equation (EE) for consumption. How does the EE compare to the one of the standard Ramsey model? Is money super-neutral here? Why or why not?

b. Now introduce money in utility in a non-separable way and assume that money and consumption are complements, utility-wise (the cross partial second derivative of utility is positive). Is money super-neutral here? Why or why not? Explain.

6. The Money-in-Utility (MIU) monetary model and the IS-LM model

Drop capital from the model and assume money enters utility in an additively separable way. Specify the market clearing conditions in the market for: (i) goods, (ii) money; (iii) bonds.

a. Show that the EE for consumption implies the existence of a forward-looking IS-type equation (i.e., a mathematical expression in which real GDP, y, and the real interest rate, r, are inversely related). How is the IS equation implied by the EE different from the standard IS equation presented in intermediate macro textbooks?

b. Show that the first order condition for money demand coupled with the market clearing condition in the market for goods, the market clearing condition in the market for bonds, and the market clearing condition in the market for money jointly imply an LM-type equation (i.e., a mathematical relation in which money demand depends positively on y, negatively on i, the nominal interest rate)

c. Are there any conditions under which the LM equation would give rise to a vertical LM curve here? If so, what are those conditions? How effective would fiscal policy be in this case?

d. Are there any conditions under which the LM equation would give rise to a perfectly flat LM curve here? If so, what are those conditions? How effective would monetary policy be in this case?

7. The MIU model (without capital) and hyperinflations
a. Set up the baseline endowment economy MIU model with no capital and no production and show analytically why hyper-deflations cannot take place in theory (they certainly do not happen in practice).

b. How many equilibria do monetary models typically have? Why is that? Explain. What requirement is it imposed on the stability of the equilibria? Why is that?

b. Is a very low rate of money supply growth sufficient to rule out the existence of hyperinflations? Why or why not? Prove your point analytically.

c. Which are the critical Inada condition(s) that should not be imposed on the utility of money balances function if hyperinflations are to be allowed to happen? In other words, what is the critical Inada condition that rules out hyperinflations? Why? Explain.