Problem 1: IS-LM and Comparative Statics

Consider the two following behavioral equations

(1)  \( C = C(Y - T, r) \quad 0 < C_{Y-T} < 1, \quad C_r < 0 \)
(2)  \( I = I(r) \quad I' < 0 \)

(a) Use words to discuss the behavioral theory imbedded in equations (1) and (2). What are equations (1) and (2) saying about the consumption and investment behavior of people?
(b) Use equations (1) and (2) to construct an IS equation.
(c) Use words to explain what your IS equation represents.
(d) Perform the calculation necessary to find the slope of your IS curve, and then plot your IS curve in the typical space.
(e) Classify the variables of your model to make the model conform with Keynesian theory.
(f) Find the output multiplier for taxes for your Keynesian model.
(g) Use words to interpret your multiplier result obtained in (f). Include an explanation of why there is a multiplier effect.
(h) Identify the behavioral conditions that will make the multiplier effect stronger versus the conditions that will make the effect weaker.
(i) Reclassify the variables of the model so the model conforms with Classical theory.
(j) Find the interest rate multiplier for taxes for your classical model.
(k) Use words to interpret your multiplier result obtained in (j). In particular, explain why an increase in the level of taxes changes the interest rate from the Classical perspective in the direction you identify.
(l) Add the following LM curve to your IS Curve and classify the variables of your new model to conform with Keynesian theory:  \( PL(Y, r) = M; L_Y > 0, \ L_r < 0 \).
(m) Find the output multiplier for taxes for this expanded Keynesian model.
(n) Use words and an IS-LM diagram to interpret the multiplier result you obtained in (m) relative to what you obtained in (f). Include a discussion of the crowding out effect.
(o) Further extend and analyze the Keynesian version of this IS-LM model as you have time, doing the following:
   a. Add a production function where output \( Y \) depends upon labor input \( N \) to endogenize the employment level \( N \). Calculate the employment multiplier for taxes and interpret your result.
   b. Add the assumption that labor is paid the value of its marginal product in order to endogenize the price level variable \( P \). Recalculate the output multiplier for taxes for this extended model and interpret your result relative to the results obtained in (f) and (m).
Problem 2: Growth Theory

Consider the following growth theory model, where the variable definitions standard.

A1 \( Y = F(K, L) \) \( F_K > 0, F_L > 0, F_{KK} < 0, F_{LL} < 0, \lambda Y = F(\lambda K, \lambda L) \)
A2 \( S = sY, \) \( 0 < s < 1 \)
A3 \( K' = I \)
A4 \( I = S, \)
A5 \( L' = nL, \) \( n \geq 0 \)

(a) Suppose the general production function (A1) takes on the particular functional form \( Y = K^\alpha L^\beta. \) Show this Cobb-Douglas production function exhibits constant returns to scale if and only if \( \alpha + \beta = 1. \)

(b) The production function \( Y = K^\alpha L^\beta \) is presented in the levels, meaning it shows how the level of output \( Y \) depends upon the level of capital input \( K \) and the level of labor input \( L. \) Convert this relationship in the levels to a relationship in the growth rates, where you show for this production function how the growth rate of output depends upon the growth rate of capital and growth rate of labor.

(c) Use words to explain why this growth theory model is useful. In particular, comment on what you use this model to do.

(d) Reduce equations (A1) to (A5) into an intensive form growth theory model.

(e) Find the positive steady state capital to labor ratio for your model, and the associated steady state output to labor ratio. To do this, show two equations that define the steady state values you are being asked to provide, and also show the two steady state values in a diagram.

(f) Perform a comparative static analysis, showing how the steady state capital to labor ratio and steady state output to labor ratio are each affected by the savings rate \( s \) and labor growth rate \( n. \)

(g) Discuss the implications of your comparative statics results obtained in (f) for policy. What do these results indicate government should encourage versus discourage?

(h) Present a phase diagram for your intensive form model. Explain in words why your phase diagram is drawn as it is, including any calculations that help with the explanation.

(i) Prove that the positive steady state for your dynamic system is stable. (To accomplish this task, you can either use the specific Cobb-Douglas production function presented in (a), or assume the Inada Conditions hold.)

(j) Expand the model (A1) to (A5) to include endogenous technical change in a manner you choose. Recognizing you have other questions to complete for this test overall, use time as you see fit to analyze your growth model with endogenous technical change. Explain what you can learn by including endogenous technical change that you cannot explain with the simpler model (A1) to (A5).
Problem 3: Asset Pricing, Lucas’ Tree Model with CES Utility

Consider an economy with $N (>0)$ identical consumers, all of whom have infinite planning horizons, constant discount factor Beta (where $0 < \beta \equiv \frac{1}{1+\rho} < 1$ is the discount factor), and maximize the discounted sum of period-specific CES utility functions. Each consumer owns a tree, which bears edible fruit every period. The amount of fruit produced by each tree at time $t$ is $y(t)$, where $y(t)$ is a stochastic process is identical across all trees. Fruit is perishable and must be consumed in the same period it is “produced.” Since all consumers are identical, this means that $c(t) = y(t)$ for all consumers in all periods. Recall that, for a risky asset, the return from holding it between period $t$ and $t+1$ is given by: $R_{t+1} = \frac{d_{t+1} + P_{t+1}}{P_t}$ (where $d(t)$ stands for the dividends paid by the tree in terms of fruit yielded, and $P$ stands for the price of the tree.

You must:

(i) Arrive at a formula that shows the equilibrium price of a tree at time $t$

(ii) How does your formula differ from the standard one for the Lucas tree model and why? Need to explain the economic intuition in words, not just refer to the math terms.

(iii) Does your pricing formula hold regardless of the assumptions made on the stochastic process followed by $y(t)$? Explain why or why not in words.
Problem 4: The Analytics of monetary non-neutrality in the MIU model
Assume that the economy is populated by a constant number of infinitely lived families who derive utility from the only consumption good \( c \) and from the stock of real money, \( m \). The representative agent intends to maximize the following utility functional:

\[
\max \int_{0}^{\infty} e^{-\rho t} \left[ u(c(t), m(t)) \right] dt; \quad \rho > 0; \ u_c > 0, u_m > 0; \ u_{mm} < 0, u_{cc} < 0; \ u_{cm} > 0
\]

Where \( m(t) \equiv \frac{M(t)}{P(t)} \), \( M(t) \) is the nominal money supply, \( c(t) \) denotes the flow of consumption services and \( P(t) \) is the price level. All variables are non-negative.

Subject to the following constraints:

\[
\dot{a} = f(k(t)) - c(t) + v(t) - \pi m(t) \quad \text{for all } t \geq 0
\]

\[
\lim_{t \to \infty} a(t) e^{-\rho t} = 0
\]

\[
k(0) > 0, \text{and given}
\]

\[
a(t) = k(t) + m(t)
\]

Where the notation is the standard one used in class, \( v \) is the per capita transfer of money from the government, and \( \pi \) is the inflation rate. The production function is neoclassical and satisfies all Inada conditions.

The other agent is a government that prints money at rate \( \mu \equiv \frac{dm}{dt} \) and runs a balanced budget, rebating seigniorage revenues immediately to the representative agent, such that: \( v(t) = \mu m(t) \) for all \( t \geq 0 \). Also, the government (or the central bank, if you prefer) targets the nominal interest rate, \( R(t) \), so as to satisfy Fisher’s parity condition, namely: \( R(t) = f'(k(t)) + \pi(t) \) for all \( t \geq 0 \).

(a) Find the set of first order conditions. Provide the economic intuition underlying the FOCs

(b) Arrive at the following Euler equation for consumption:

\[
\frac{\dot{c}}{c} = \sigma(.) \left[ f'(k(t)) - \delta - \rho - \xi \eta \frac{R}{R} \right]
\]

Where \( R \) is the nominal interest rate, \( \frac{\dot{R}}{R} \) is the percent change in the nominal interest rate, \( \eta \) is the interest rate elasticity of money demand, and \( \xi \) is the elasticity of the marginal utility of consumption relative to money balances. Both elasticities are non-negative.

(c) Provide the intuition (= economic interpretation) of the term \( -\xi \eta \frac{R}{R} \) in equation (6) above. Hint: start from the far right and move leftward component by component. In particular, what happens if agents anticipate a future increase in the nominal interest rate?

(d) Analyze money’s super-neutrality (or lack thereof) in the present model.

(e ) What assumptions do we need to append to the second model in problem 1 above to transform it into a discrete time version of the MIU model analyzed in this problem?