Question 1: IS-LM and Comparative Statics

Consider the following equations

(1) \( C = C(Y - T) \quad 0 < C' < 1 \)
(2) \( I = I(r, Y) \quad I_r < 0, \ I_Y > 0 \)
(3) \( Y = C + I + G \)
(4) \( Y = C + S + T \)

(a) Use words to explain the difference between investment \( I \) consumption \( C \), and explain why economists find it useful to separate investment spending from consumption spending.

(b) What are the restrictions \( I_r < 0 \) and \( I_Y > 0 \) saying about investment behavior, if \( r \) is the economy’s real interest rate level and \( Y \) is the economy’s output level?

(c) Draw a schematic of the macro economy which illustrates the flows presented in equations (3) and (4). Include in your diagram a household sector, a firm sector, a government sector, and a capital market. Show how equations (3) and (4) indicate these different economic entities are related.

(d) Use equations (1)-(4) to construct an IS equation.

(e) Use words to explain what your IS equation represents relative to the schematic drawn to answer (c).

(f) Classify the variables of the model (1)-(4) to make the model conform with Keynesian theory.

(g) Find the output multiplier for government purchases for your Keynesian model.

(h) Use words to interpret your multiplier result obtained in (g). Include an explanation of why there is a multiplier effect.

(i) Identify the behavioral conditions that will make the multiplier effect stronger versus the conditions that will make the effect weaker.

(j) Reclassify the variables of the model so the model conforms with Classical theory.

(k) Find the interest rate multiplier for government purchases for your classical model.

(l) Use words to interpret your multiplier result obtained in (j). In particular, use the Classical perspective to explain why an increase in the level of government purchases changes the interest rate in the direction you identify.

(m) Add the following LM curve to your IS Curve and classify the variables of your new model to conform with Keynesian theory: \( P_L(Y, r) = M; L_Y > 0, L_r < 0 \).

(n) Find the output multiplier for government purchases for this expanded Keynesian model.

(o) Use words and an IS-LM diagram to interpret the multiplier result you obtained in (n) relative to what you obtained in (g). Include a discussion of the crowding out effect.

(p) As you have time, construct and analyze a Classical version of the IS-LM model, doing the following:

   i. Add a production function where output \( Y \) depends upon labor input \( N \).
   ii. Add the assumption that labor is paid the value of its marginal product
   iii. Add a labor supply function
   iv. Add the assumption that the labor market clears.
   v. Again, derive the output multiplier for government purchases, and interpret your result.
   vi. Derive the real interest rate multiplier for government purchases, and interpret your result.

   vii. Using words, explain why the level of government purchases does not influence the output level but does influence the real interest rate level from the Classical perspective.
Question 2: Growth Theory

Consider the general research and development growth model, where the variable definitions are standard.

(1) \( Y = \left[ 1 - a_K \right] K^\alpha \left[ A \left[ 1 - a_L \right] L \right]^{1 - \alpha} \), \( 0 < \alpha < 1 \)
(2) \( A' = B \left[ a_K K \right] \beta \left[ a_L L \right] \gamma A^\theta \), \( 0 < \theta < 1 \)
(3) \( K' = sY \), \( 0 < s < 1 \)
(4) \( L' = nL \), \( n \geq 0 \)

Endogenous (4): \( Y, K', A', L' \)
Predetermined (3): \( K, A, L \)
Exogenous (9): \( B, s, n, \alpha, \beta, \gamma, \theta, a_K, a_L \)

(a) Show the production function (1) exhibits constant returns to scale.
(b) Treating \( a_K, a_L, \text{ and } \alpha \) as constants, convert the production function (1) from a relationship “in the levels” to a relationship “in the growth rates.” That is, show how the growth rate of output is related to the growth rates of capital, labor, and technology if production occurs as specified by the production function (1).
(c) Use words to explain why this growth theory model is called an “endogenous” growth theory model.
(d) Transform the model (1)-(4) into a model that includes two endogenous variables: \( g_K' \) and \( g_A' \), where \( g_K' = d[g_K]/dt \), \( g_K = K'/K \), \( g_A' = d[g_A]/dt \), and \( g_A = A'/A \). Classify all of the variables in your resulting two equation model as either being endogenous, exogenous, or predetermined.
(e) Using words only, explain the meaning of the terms “endogenous,” “exogenous,” and “predetermined” using variables in this model to illustrate. Write your explanation in a way that expresses what you can learn by analyzing this model.
(f) Find the positive steady state values for \( g_K \) and \( g_A \) for this model.
(g) Assuming the rate of population growth \( n \) is positive, show how \( g_Y, g_K \) and \( g_A \) are related to each other in the steady state. Comment on whether or not the relationships you derive are consistent with data on the U.S. economy long term.
(h) Present the \( g_K' = 0 \) and \( g_A' = 0 \) null clines in a coordinate plane, where you plot \( g_A \) along the horizontal axis and \( g_K \) along the vertical axis. Assume \( \beta + \theta < 1 \).
(i) Calculate the direction of movement in each of the four different areas of the phase diagram you constructed for (h). Once you have done that, take one point in each of the four areas and plot the trajectory the economy would follow if the economy started from that point.
(j) Using words, explain what you learn from the phase diagram analysis you completed in (i).
Question 3 (20 points): Asset Pricing, Lucas’ Tree Model with CES Utility

Consider an economy with \( N (> 0) \) identical consumers, all of whom have infinite planning horizons, constant discount factor Beta (where \( 0 < \beta \equiv \frac{1}{1+\rho} < 1 \) is the discount factor), and maximize the discounted sum of period-specific CES utility functions. Each consumer owns a tree, which bears edible fruit every period. The amount of fruit produced by each tree at time \( t \) is \( y(t) \), where \( y(t) \) is a stochastic process is identical across all trees. Fruit is perishable and must be consumed in the same period it is “produced.” Since all consumers are identical, this means that \( c(t) = y(t) \) for all consumers in all periods. Recall that, for a risky asset, the return from holding it between period \( t \) and \( t+1 \) is given by:\[ R_{t+1} = \frac{d_{t+1} + P_{t+1}}{P_t} \] (where \( d(t) \) stands for the dividends paid by the tree in terms of fruit yielded, and \( P \) stands for the price of the tree.

You must:

(i) Arrive at a formula that shows the equilibrium price of a tree at time \( t \)

(ii) How does your formula differ from the standard one for the Lucas tree model and why? Need to explain the economic intuition in words, not just refer to the math terms.

(iii) Does your pricing formula hold regardless of the assumptions made on the stochastic process followed by \( y(t) \)? Explain why or why not in words.

Question 4 (30 points) : The Analytics of monetary non-neutrality in the MIU model

Assume that the economy is populated by a constant number of infinitely lived families who derive utility from the only consumption good \( c \) and from the stock of real money, \( m \). The representative agent intends to maximize the following utility functional:

\[
\max \int_0^\infty e^{-\rho t} \left[ u(c(t), m(t)) \right] dt; \quad \rho > 0; \quad u_c > 0; \quad u_m > 0; \quad u_{mm} < 0; \quad u_{cc} < 0; \quad u_{cm} > 0
\]

Where \( m(t) \equiv \frac{M(t)}{P(t)} \), \( M(t) \) is the nominal money supply, \( c(t) \) denotes the flow of consumption services and \( P(t) \) is the price level. All variables are non-negative.

Subject to the following constraints:

\[
\begin{align*}
(2) \quad & \dot{a} = f(k(t)) - c(t) + v(t) - \pi m(t) \quad \text{for all } t \geq 0 \\
(3) \quad & \lim_{t \to \infty} a_t e^{-\rho t} = 0 \\
(4) \quad & k(0) > 0, \text{ and given} \\
(5) \quad & a(t) = k(t) + m(t)
\end{align*}
\]
Where the notation is the standard one used in class, \( v \) is the per capita transfer of money from the government, and \( \pi \) is the inflation rate. The production function is neoclassical and satisfies all Inada conditions.

The other agent is a government that prints money at rate \( \mu \equiv \frac{dm}{dt} \) and runs a balanced budget, rebating seigniorage revenues immediately to the representative agent, such that: \( v(t) = \mu m(t) \) for all \( t \geq 0 \). Also, the government (or the central bank, if you prefer) targets the nominal interest rate, \( R(t) \), so as to satisfy Fisher’s parity condition, namely: \( R(t) = f'(k(t)) + \pi(t) \) for all \( t \geq 0 \).

(a) Find the set of first order conditions. Provide the economic intuition underlying the FOCs

(b) Arrive at the following Euler equation for consumption:

\[
\frac{\dot{c}}{c} = \sigma(\cdot)[f'(k(t)) - \delta - \rho - \xi \eta \frac{\dot{R}}{R}]
\]

Where \( R \) is the nominal interest rate, \( \frac{\dot{R}}{R} \) is the percent change in the nominal interest rate, \( \eta \) is the interest rate elasticity of money demand, and \( \xi \) is the elasticity of the marginal utility of consumption relative to money balances. Both elasticities are non-negative.

(c) Provide the intuition (= economic interpretation) of the term \(-\xi \eta \frac{\dot{R}}{R}\) in equation (6) above. Question: What happens if agents anticipate a future increase in the nominal interest rate?

(d) Analyze money’s super-neutrality (or lack thereof) in the present model. Explain your equations with words, providing the economic interpretation of your results.

(e ) Suppose you were to be provided with data for money demand in real terms (\( m = M/P \)), nominal and real interest rate data, real GDP data, and consumption data for a number of years. Question: How would you test for the empirical validity of the MIU versus the CIA models of money?

You need to: (i) set up the empirical model(s); (ii) clearly spell out \( H(0) \) and \( H(1) \) for each case; (iii) explain what would make a particular model invalid in light of the estimated coefficients versus the models’ predictions (i.e., \( H0 \) and \( H1 \)), given the sample data.