Macroeconomic Theory Comprehensive Exam
Winter 2014

Question 1

a. Consider the consumption function \( C = C(Y - T, r) \), where \( Y \) is the level of real income, \( T \) is the real level of taxation, and \( r \) is the real interest rate level. Take the total differential of this function, and write a sentence explaining in words the meaning of the partial derivatives in your result.

b. Propose restrictions on the partial derivatives present in the result you obtained for the last question. Write a sentence explaining in words why each restriction you propose is behaviorally sensible.

c. Suppose income \( Y \) from the sale of output is split between consumption \( C \) and saving \( S \) so \( Y = C + S \). Use this constraint and the consumption function of part (a) to derive a savings function. Totally differential that savings function, and explain in words what your results is saying about how the level of savings is affected by other factors.

d. Assume \( G \) denotes the real level of government purchase and ignore any foreign transactions, so \( AD = C + I + G \) represents the real aggregate demand level. Further, assume the real level of investment depends only upon the real interest level, so \( I = I(r) \), with \( I'(r) < 0 \). Present an IS equation for this economy. Find the slope of this IS curve, and use your calculation to appropriately plot the IS curve. Also, explain (in economic terms, not in pure mathematical terms) when the IS curve will have a less steep slope.

e. To your IS equation, add the LM equation \( M = PL(r, Y) \) with the assumptions \( L_r < 0 \) and \( L_Y > 0 \), so together your IS equation and LM equation represent a two equation system. Assume the economy behaves according to traditional Keynesian assumptions. Obtain four multipliers: \( dY/dT, dY/dM, dr/dT, dr/dM \).

f. Use your multiplier results from the last question to discuss expansionary monetary policy versus expansionary fiscal policy.
   a. To start, show the impact of expansionary monetary policy in one IS-LM diagram and expansionary fiscal policy in another.
   b. Then, explain why the effectiveness of expansionary fiscal policy suffers from crowding out while expansionary monetary policy does not.
   c. Finally, explain when fiscal policy will be especially ineffective because of crowding out. In answering, using both the IS-LM curves and your the multiplier calculations.

g. If you have additional time, reclassify the variables of your two equation IS-LM model so the model is classical, derive one or more multipliers, and use your results to explain how the classical perspective of how the economy works differs from the Keynesian perspective.
Question 2

Consider the following growth model:

\( \Lambda 1 \quad Y = F(K, AL) \quad \quad \Lambda Y = F(\lambda K, \lambda AL) \)

\( \Lambda 2 \quad Y = C + I + G \)

\( \Lambda 3 \quad S = sY \quad 0 < s < 1 \)

\( \Lambda 4 \quad K' = I - \delta K \quad 0 < \delta < 1 \)

\( \Lambda 5 \quad T = tY \quad 0 < t < 1 \)

\( \Lambda 6 \quad I = S - [G - T] \),

\( \Lambda 7 \quad L_t = L_0 e^{nt} \),

\( \Lambda 8 \quad A_t = A_0 e^{nt} \),

Definitions:

\( Y \quad \text{Output level} \)

\( K \quad \text{Capital level} \quad K' = dK/dt \quad \text{Change in capital} \)

\( L \quad \text{Labor level} \quad G \quad \text{Government purchases level} \)

\( S \quad \text{Savings level} \quad T \quad \text{Taxation level} \)

\( I \quad \text{Investment level} \quad s \quad \text{Savings rate} \)

\( A \quad \text{Technology Level} \quad n \quad \text{Labor growth rate} \)

\( \mu \quad \text{Technology growth rate} \quad \delta \quad \text{Capital depreciation rate} \)

(a) Suppose the production function is Cobb-Douglas so \( F(K, AL) = K^a [AL]^{1-a} \). Demonstrate your production function exhibits constant returns to scale.

(b) Transform the Cobb-Douglas production function into a growth rates model, where the growth rate of output depends upon the growth rate of labor, the growth rate of capital and the growth rate of technology. Explain in words what your calculation is telling you.

(c) Explain in words why equation A4 sensibly represents how capital might accumulate.

(d) Reduce the growth model presented as equations A1-A8 to an intensive form model with only one state variable \( k \), where \( k = K/AL \). Classify the variables of your intensive form model.

(e) Obtain the single equation that defines the steady state for the model: Use a diagram to show that this model will have no steady state if the government purchases level “per capita” \( g = G/AL \) is large enough, two steady states if \( g \) is positive but small enough, and one steady state if \( g \) is positive and just the right level.

(f) Assume there are two steady states. Derive the comparative static multiplier \( \frac{dk}{dg} \). Use your result to show that one of the two steady state capital levels will increase when government purchases more, while the other steady state capital level will decrease when government purchases more.

(g) Construct a phase diagram for the version of this model assuming there are two steady states. Perform the calculations necessary to demonstrate any curve you drawn in the diagram has the shape you assume. Using the diagram, discuss the stability of the two steady states. If you are able, add any mathematical analysis you can to complement the diagrammatic analysis, which will help analyze the stability of the two steady states.

(h) If you have time, examine how the steady state per capita consumption level \( c = C/AL \) depends upon the per capital level of government spending \( g \) and the tax rate \( t \). Going further, you could seek to identify “golden rule” levels for the parameters \( g \) and \( t \). (Hint: The golden rule levels will be where the sum \( c + g \) is maximized, because people benefit from both private and public goods.)
**Question 3**

A competitive firm chooses the amount of investment in capital equipment (and the amount of labor) in order to maximize the present discounted value of its cash flow.

\[
\text{PDV (CF)} = \int_{0}^{\infty} e^{-rt} [F(K_t, L_t) - wL_t - p_t I_t - C(I, K)] dt
\]

Where all the variables retain the meaning given in class and in all standard textbooks (Takayama, Romer, for ex.):

\[C_{I}(. > 0; C_{I_t}(. > 0 \text{ for all } I > 0; \ C(I = 0) = 0; \ C_{I}(I = 0) \geq 0}\]

\[C_{K}(. > 0; C_{K}(. > 0 \text{ for all } K > 0; \ C(K = 0) = 0; \ C_{K}(K = 0) \geq 0 \ C_{IK}(. > 0}\]

The maximization is subject to the following constraints:

\[\dot{K} + \delta K_t = I_t; \ 0 < \delta < 1 \text{ (Net investment plus replacement investment is equal to total or gross investment).}\]

\[K(0) > 0 \text{ and given}\]

\[K(t) \geq 0 \text{ for all } t\]

Relevant TVC condition

You must:

1. Declare the control and state variables, form the Hamiltonian (current value is recommended) and find the first order conditions which are necessary and sufficient for a maximum. Provide economic interpretations for the first order conditions relative to investment: (explain the meaning of lambda and the reason why it is different from p) and labor.

2. Using your system of differential equations and with the aid of phase diagrams, analyze the effects of:

   1. A permanent, unanticipated, current increase in \( r \), the real interest rate
   2. A temporary, future, fully anticipated increase in \( r \).

3. Add taxes/subsidies into the model. Imagine that the purchase price of investment goods is modified in such a way that the effective price of purchasing investment goods includes not only
the nominal or dollar price of buying machinery but also the tax/subsidy paid per unit every time
equipment is purchased. For example, \( p(E) = p/(1-\tau) \).
You must:
1. Using the modified system of differential equations and a phase diagram, analyze the
effect of a permanent, unanticipated, current decrease in \( \tau \).
2. Using the modified system of differential equations and a phase diagram, analyze the
effect of a temporary, anticipated, future decrease in \( \tau \).
3. Which of the two tax policies (permanent vs. temporary tax cut) has a bigger effect to
boost the economy and why?

**Question 4: The Money-in-production model**

Present the standard money-in-production model. You must:

1) Present the structural form of the model (the initial equations and assumptions),

2) Form the Hamiltonian, and calculate the first order necessary and sufficient conditions for a
maximum,

3) Combine equations to arrive to the canonical or characteristic differential equation in real
money balances that completely describes the dynamics of the system

4) Analyze the stability conditions that are sufficient to guarantee that monetary policy is able to
pin down a unique price level. In other words, what conditions applied to the canonical
differential equation in real money balances guarantee that at the steady state with positive real
money balances the differential equation is locally unstable?

5) Does this model display super-neutrality? Why or why not? Explain. What equation/equations
clearly show the result you advocate?