

Math 442/642 Spring 2009

Differential Topology

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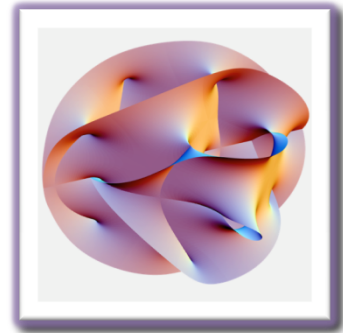
The gateway to “higher mathematics” is through the calculus of the infinitesimal as invented by Newton and Leibnitz over 300 years ago. Their ideas led to the definitions of continuity, differentiability and integrability of real functions, the foundations of modern mathematics.



Over the many intervening years, these concepts have found ever increased maturity in ever more sophisticated mathematical theories. For instance, *Measure Theory* has been invented to properly address some of the shortcomings of the Riemann integral; *Topology* in turn was born to put continuity on a proper footing, one independent of the vector space structure of

Euclidean space.

Differential Topology is the study of differentiable functions on *smooth manifolds* – spaces that are vast generalizations of Euclidean space. Smooth manifolds appear in many areas of mathematics and modern physics. Two famous examples of are those of Einstein’s space-time continuum and Calabi-Yau manifolds (shown in the pictures) arising in string theory.



Differential forms have led to a deeper understanding of already known physical concepts. For example, Maxwell’s famous 4 equations of electro-magnetism, when expressed in the language of differential forms, simply become $dF=0$ and $d^*F=J$ (see Wikipedia’s nice entry on Maxwell’s equations).

This course is an introduction to these exciting concepts and will cover the basics of smooth manifold theory and differential forms. We will use the text

John Lee: “Introduction to smooth manifolds”, Springer Verlag 2002