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Mathematics Assessment

For

Mathematics before Calculus

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Introduction

We are in the midst of a dynamic and exciting time in mathematics education in the United States of America. Our educational system has been reorganizing at all grade levels. This is true of the college level mathematics, lower-division university mathematics, high school mathematics, and college-prep mathematics for pre-college students.

The mathematics reform movement created the need to establish **Standards** that reflect the changes in math education. Although many departments of mathematics at colleges and universities are not in complete agreement as to how much content to change, which pedagogues are necessary, or how much technology to incorporate into particular courses, one can see from topics at professional conferences, educational journals and textbooks that the reform movement has had considerable impact on mathematics education. “It has been just a decade since A Nation at Risk took aim and fired its salvos at the American educational system, initiating an intense reexamination of the quality of teaching and learning at all levels of education”. (Angelo, p. xiii, 1993)

Mathematics, probably more than any other discipline is under scrutiny by the public. Consequently, national Standards have been established for mathematics education at all levels. The Standards established by the professional educational mathematics organizations establish guidelines and objectives for mathematics education.

The challenge now is to synthesize and incorporate these exciting **Standards** into our courses. We then must assess student outcomes based on those standards. How can we show student achievement of those standards?

As we implement these Standards into the curriculum and into our teaching it will be also necessary to assess their effectiveness. Assessment should be expanded to measure the success of those changes based on the Standards which suggest that "faculty must help their students think critically, learn how to learn, and find motivation for the study of mathematics in appreciation of its power and usefulness" (p. xii, AMATYC, 1995). How do we address these objectives in our classes?

This workbook is intended to address changes in learning objectives and their related assessments. The “Standards” referred to in this book are the AMATYC **Standards** from the *Crossroads in Mathematics*. These Standards are also congruent with many of the Standards for mathematics education from NCTM for grades 9 – 12.

The Standards

The Standards are in three categories:

1. **Standards for Intellectual Development**
2. **Standards for Content**
3. **Standards for Pedagogy**

The following will be descriptions of the **Standards for Intellectual Development** and **Pedagogy**.

Descriptions: (The following descriptions of the **Standards** are taken in all or excerpted from *The Crossroads . pp. 9 - 17).* Within each of the standards, certain words or phrases will be underlined. The underlining indicates an assessment that is addressed in this document.

Standards for Intellectual Development address modalities of student thinking and reasoning. ***They will be measured by student outcomes.***

STANDARDS FOR INTELLICTUAL DEVELOPMENT

Standard 1-1: Problem Solving
Students will use problem-solving strategies that require persistence, the ability to recognize inappropriate assumptions and intellectual risk taking rather than simple procedural approaches. The strategies should include posing questions; organizing information; drawing diagrams; analyzing situations through trial and error graphing and modeling; and drawing conclusions by translating, illustrating, and verifying results. The student should be able to communicate and interpret their results.

Standard 1-2: Modeling .
Students will participate in the mathematical modeling of situations from the world around them and use the models to make predications and informed decisions. Swetz *(1991)* describes the modeling process as "(1) identifying the problem, and including the conditions and constraints under which it exists; (2) interpreting the problem mathematically; (3) employing the theories and tools of mathematics to obtain a solution to the problem; (4) testing and interpreting the solution in the context of the problem; and (5) refining the solution techniques to obtain a *better* answer to the problem. Students may be asked to evaluate previously develop models by addressing questions such as: (1) does the model behave as intended, i.e. do the equations fit the assumptions of the model? (2) how well does the model agree with the real world it is suppose to represent? (3) does the model perform well on a data set different from the one for which it was developed?

Standard 1-3: Reasoning
Students will regularly apply inductive and deductive reasoning techniques to build convincing mathematical arguments. They will develop conjectures on the basis of past experiences and intuition and test these conjectures by using logic and deductive and inductive proof, by framing examples and counterexamples, and by probabilistic and statistical reasoning. Students will judge the validity of mathematical arguments and draw appropriate conclusion.

Standard 1-4: Connecting with Other Disciplines
Course content must include topics developed since the eighteenth century. Students need to research sources other than standard mathematics textbooks to determine how mathematics provides a language for the sciences; plays a role in art, music, literature; is applied to economics; is used in business and manufacturing; and has had an impact on history.

Standard 1-5: Communicating
Students will communicate mathematical ideas and procedures using appropriate mathematical vocabulary and notation. Students will learn to read and listen to mathematical presentations and arguments with understanding. Students will learn to speak and write about mathematics, especially of real applications.

Standard 1-6: Using Technology
Students will develop the ability to use technology to aid in the understanding of mathematical principles. Students will use technology naturally and routinely as a tool to aid in the solution of realistic mathematical problems.

Standard 1-7: Developing Mathematical Power
All students will have the opportunities to be successful in doing meaningful mathematics that fosters self-confidence and persistence. They will engage in solving problems that do not have unique answers but, rather, provide experiences that develop the ability to conduct independent explorations. At the same time, they will learn to abstract mathematical principles in order to promote transfer of problem-solving strategies among a variety of contexts (Druuckman & Bjork, 1994) and to better appreciate mathematics as a discipline. Furthermore, they will develop an awareness of careers in mathematics and related disciplines and have a vision of themselves using mathematics effectively in their chosen fields.

STANDARDS FOR PEDAGOGY

The theory of educational constructivism is based on the premise that knowledge cannot be “given” to students. Rather, it is something they must construct for themselves. The standards for pedagogy are from a constructivist model. They use

instructional strategies that provide for student activity and student-constructed knowledge.

Standard P-1: Teaching with Technology
Faculty will use dynamic computer software to aid students in learning mathematics concepts. Faculty will model appropriate use of technology as tools to solve mathematical problems. Faculty will use technology as a medium of instruction. (note: all instructional objectives)

Standard P-2: Interactive and Collaborative Learning
Mathematics faculty will foster interactive learning through the following types of experiences in college classrooms: cooperative learning (Crocker, 1992; Becker & Pence, 1994); oral and written reports presented individually or in groups; writing in journals; open ended projects; alternative assessment strategies such as essay questions and portfolios (Leitzel, 1991; NCTM, 1991). (note: all instructional objectives)

Standard P-3: Connecting with Other Experiences
Mathematics faculty will actively involve students in meaningful. mathematics problems that build on their experiences, focus on broad mathematical themes, and build connections within branches of mathematics and between mathematics and other disciplines so that student will view mathematics as a connected whole relevant to their lives.
(note: all instructional objectives)

Standard P-4: Multiple Approaches
Mathematics faculty will model the use of multiple approaches numerical, graphical, symbolic, and verbal- to help students learn a variety of techniques for solving problems. (note: all instructional objectives)

Standard P-5: Experiencing Mathematics
Mathematics faculty will provide learning activities, including projects and apprenticeships, that promote independent thinking and require sustained effort and time so that students will have the confidence to access and use needed mathematics and other technical information independently, to form conjectures from an array of specific examples, and to draw conclusions from general principles.

Mathematics faculty will assign open-ended classroom and laboratory projects. In addition, they will help their institutions form partnerships with area businesses and industry to develop opportunities for students to have realistic career experiences (Reich, 1993).
(note: instructional objectives)

What are Objectives?

As the **Standards** direct educators to new instructional objectives, assessment of student outcomes are necessary to measure how closely we come to accomplishing those objectives.

The outcome of instruction should be measurable. These measurable outcomes are referred to as student outcomes, behavioral objectives, learning objectives, or performance objectives. We define student outcomes as a statement of a capability or skill of the learner. It can be observed as a performance. “This performance may be put into measurable terms by answering the question: *What will the learner be able to do after instruction that s/he couldn't do before instruction*?" (Gagne', et al. 1988, p. 121). (author’s italics)
"An instructional objective is a desired outcome of learning that is expressed in terms of observable behavior or performance of the learners. The learners should be able to do something that they could not do prior to the learning experience provided by the teacher. **The observed behaviors at the end of the** lesson are going to be either new behaviors or extensions of existing behaviors. An instructional objective obviously will not measure all of the possible outcomes of a learning experience, but it should measure those outcomes specifically desired by the teacher" (Montague, p.3, 1987). Therefore, what we wish to accomplish in class (instructional objectives) are measured (assessed) by what the students can now do (student outcomes) after a lesson.

The Crossroads direct educators to look at instructional objectives in the broad category of **Standards for Intellectual Development**. They include **Standards** for:

1. Problem Solving
2. Modeling
3. Reasoning
4. Connecting with Other Disciplines
5. Communicating
6. Using Technology
7. Developing Mathematical Power

Assessments for these instructional objectives will be measured in terms of student outcomes.

Student outcomes can be thought of in three broad domains: cognitive , affective and psychomotor. In this discussion we will concern ourselves with the first two domains. Cognitive outcomes have to do with knowledge and the use of higher order mental processes. Affective outcomes have to do with the student's feelings, attitudes, values, beliefs, self-concept, aspirations, and social and interpersonal relationships (Astin, 1991, p. 43). The cognitive domains are assessed in this workbook. We can “measure the cognitive domain through behavioral data reflecting the student's observable activities” (Astin, 1991, p. 44).

Affective learning refers to the values, attitudes, feelings and appreciation that may result from the learning experience (Krathwohl, 1964). Although assessing these objectives can be difficult, the **Standards** do include affective outcomes. Assessments for Affective outcomes are informal and are listed in the Appendix.

As you read through **The Standards**, you will see that the level of cognitive learning that is desired is at the top half of the hierarchy. Although you may desire to ask questions that refer to lower levels of cognitive learning for content, the objectives for Intellectual Development are at the higher levels of cognitive complexity.

BLOOM'S TAXONOMY OF LEARNING

Cognitive learning involves knowledge and understanding. It may include recall, comprehension, application, analysis, synthesis or evaluation of knowledge. (Montague, p. 5, 1987) These classifications are similar to those in Bloom's taxonomy of Learning. Benjamin Bloom (Bloom, 1956) suggested a classification system for different kinds of cognitive learning. The six categories are knowledge, comprehension, application, analysis, synthesis, and evaluation. Mathematics educators have an intuitive feel for what are "easy, medium and difficult" problems or tasks in their courses. Blooms' Taxonomy allows us to describe specifically why a problem or task may be more difficult than another.

Following is a brief description of Bloom's Taxonomy of Learning. Benjamin Bloom created this heirachery of learning in the mid 1950's. Its purpose is to be able to relate tasks, questions, problems, etc. to each other. It allows educators to indicate the level of cognitive complexity for a task. The levels are described from lowest to highest. The highest level, Evaluation, involves the most cognitive complexity.

Following is a brief description of each:

BLOOM'S TAXONOMY OF LEARNING

1. Knowledge: The ability to remember facts in a form similar to that in which they were presented.
2. Comprehension: The ability to translate some knowledge into your own words.
3. Application: The ability to apply learning to new situations.
4. Analysis: The ability to break down a situation into its component parts and to detect relationships between the parts, or a part to the whole.
5. Synthesis: The ability to organize or assemble parts to form a new whole.
6. Evaluation: The ability to make judgments based on identified criteria or standards. (Montague, p. 5, 1987)

The **Standards** ask for higher cognitive levels of learning. Therefor assessments must be appropriate to measure the higher cognitive levels in Bloom's Taxonomy. The level of the learning and of its measure will be indicated in each assessment.

True Colors™

There are several systems available for typing personality. Probably the most widely known is that of the Myers-Briggs Personality Inventory, developed in 1932. This inventory was based on the work of Carl Jung, an eminent psychoanalyst.

Personalities can generally be typed into 4 categories. Of the several personality typing systems, all have defined 4 categories. Kolb, Lawrence, Hunt, McCarthy (McCarthy, p.34, 1987) and *Lowry* name a few of the researchers.

Don Lowery, a former teacher in Irvine, CA, developed **True Colors™** in 1979. The system is based on the MyersBriggs Personality Type Indicator and the research of David Keirsey as communicated from the book, *Please Understand Me.*

Why would it be important to understand personality types in a math classroom? Does the instructor need to know his or her personality type? I will address these important questions.

Students come to us with different learning styles. That means that some students will enjoy our classes and feel like it was one of their best learning experiences, while others will not experience the same level of excitement about the learning. This is often a reflection of the match between the teaching style of the educator and the learning style of the student. And those styles are a result of the personality type of the teacher and the student.

Students will learn best and be able to express their learning most accurately using processes that match their personality. This, however, does not imply that students will not learn if their personality is different from the instructors, or the pedagogy is not their preferred style. It does imply that some students will perform well in some learning environments, while others may perform poorly in the same environment. We have all seen that. For that matter, we have probably experienced this ourselves. There are mathematics professors we have truly enjoyed and felt as though we had a tremendous learning experience in their particular class. We probably enjoyed the assessment procedures and felt as though we did our best in those classes. The same is true for our students. And in other classes, we struggled to try to understand.

In each classroom, we have a mixture of learning styles. We also have our teaching style, which to a large extent is an expression of our personality. When we assess student learning, it is possible to vary the types of assessments to “appeal” to the variety of learners. The assessments that are presented in this book are varied in style

to let each personality shine in their best light. Not every assessment will appeal to every student, but some assessments will appeal to all students. In this way, we can see the best performance by all of our students some of the time.

Classifying Learners

True Colors™ classifies learners into four colors based on how they prefer to learn:

Green learners prefer to work independently. They are:

- Logical
- Theoretical
- Curious
- Conceptual
- Driven to understand
- Learn best independently
- Need to be immediately challenged

Gold Learners do best when content is structured. They:

• prefer useful subjects
• thrive on routine and orderliness
• are punctual and dependable
• think problems through before making a decision
• respect rules
• have a strong sense of right and wrong
• respect the institution of the school

Blue learners feel best in interactive atmospheres. They are:

• verbal and good with languages
• imaginative and abstract
• social and work best in a group setting
• sensitive to rejection and to conflicts with teachers
• dramatic

Orange learners perform well in competition. They:

• learn by doing
• like tools
• are impulsive
• are physical like being on stage are competitive like immediate results are a hands-on type person

Classifying Teachers

True Colors™ also describes teachers' behavior based on their color. According to **True Colors**™:

Blue teachers are student oriented-

- They use their imagination as a teaching tool
- They involve their students in the learning process
- They seek to create harmony in the classroom
- They are best when they can use individualized instruction
- They are concerned with the whole student
- They operate a democratic classroom

Green teachers are subject oriented –

- They seek answers to natures enigmas and encourage students to do the same
- They are interested in the development of their students' intelligence
- They enjoy inspiring students to stretch their intellects
- They present material in a logical manner
- They use cause and effect approaches to develop the reasoning ability of their students

Gold teachers are institution oriented –

- They have well-established classroom routine
- They foster and reward delegated responsibility
- They encourage team effort
- They structure the learning process in a step-by-step fashion
- They preserve and transmit cultural heritage within the institution
- They are firm and fair with discipline

Orange teachers are action oriented –

- They are exciting innovators
- Their teaching style is dynamic and unplanned
- They provide a variety of action experiences
- They teach in the here and now
- They deal with concrete problems
- Their direction is dramatic and spontaneous

All 4 personality types will be present in our classrooms, although some types will be

better represented. According to a two-year study conducted in 1988 and 1989 by the Advertising Media Partnership for a Drug-Free America, college students tend to be either **gold** or **blue** personality types in the **True Colors™** system.

The results of the 9300 college students
and adults surveyed are as follows:

True Colors™ of 9300 College Students and Adults

	College Students	Adults
Blue	30%	23%
Green	17%	13%
Gold	38%	53%
Orange	16%	12%

(from True Colors™ Resource and Reference Guide).
This information captures our attention because our personality color will determine our natural teaching style. If we are sensitive to the fact that we have different "colors" in our classes, we can alternate our teaching and assessment practices to better reach all students. Therefore the assessments that follow in this text are varied. Some techniques will be more appealing for certain color types. That same assessment may not appeal to other color types. Each assessment is "color coded". By varying the "color" of the assessments, you will reach more students effectively. It may be important to remind ourselves that we may not be able to maximize the learning experience for all students all the time. We can however, by varying our assessments and teaching styles, reach all of the students some of the time. And that is a pretty good thing.

Assessment

The overall concept of classroom assessment is now one of a continuous and dynamic process. **The National Council of Teachers of Mathematics** is the largest professional organization of mathematics teachers for grades K – 12. The document *Curriculum and Evaluation Standards* compiled by NCTM addresses standards in mathematics education for those grade levels. Many of the NCTM Standards for grades
9 –12 carry the same intent as the Standards in *The Crossroads* for college mathematics before calculus. "One conception of assessment portrayed in the *Curriculum and Evaluation Standards* (written by the National Council of Teachers of Mathematics) treats assessment as a process in which teacher tries to understand the meaning that students assign to the ideas that are covered in the dialogues that take place between teachers and students during the teaching process. One of the implications of trying to understand the meaning that students assign to ideas is that

assessment is treated as though it were a continuous and dynamic process, rather than static testing at specific points in time. The *Standards* make three specific points concerning the ways that assessment can be changed so that it is more adaptable to this dynamic view of assigning meaning to students' ideas and so that assessment enables student to gain the knowledge of mathematics that is envisioned in the *Standards*: Student assessment should be integral to instruction; multiple assessment methods should be used; and all aspects of mathematical knowledge and it connections to other branches of knowledge should be addressed" (Webb, NCTM Yearbook, 1993, p.2)

Assessment in the college mathematics classroom can no longer be thought of as quizzes and homework alone. The assessment themselves must also address the learning and teaching objectives from **The Standards** and give students useful feedback as to their level of mastery of those objectives. To some extent, the course must be flexible enough to accommodate necessary changes based on the assessment results. Assessment has grown to have a fuller meaning.

"Rapid changes are taking place in conceptions of assessment that focus on the mathematical knowledge of students. A narrow conception of assessment as paper-and-pencil tests that require students to produce an expected-and generally numerical-answer is being challenged by an eclectic view of assessment that draws on many methods to ascertain individual students' knowledge of mathematics. The impetus for this changing view of assessment derives a strong demand that students know mathematics and be able to use mathematics in the changeable world that these students will face during their lifetimes" (Webb, NCTM Yearbook, 1993, p.1)

Assessment can have an informative and supportive intention for faculty and students. However, "*assessment results are of most value when they shed light on the causal connections between educational practice and educational outcomes"*. (Austin, pp. xii, 1991).

Teachers need a continuous flow of information on student learning and their progress in that process. "Assessment, and its results, is not-and should not be interpreted as-the end of educational experiences; instead, it is a means to achieve educational goals" (Webb, p.1, NCTM 1993).

Using assessment as ongoing data collection about student learning make educators into statisticians and educational researchers. "Faculty have an exceptional opportunity to use their classrooms as laboratories for the study of learning and through such study to develop a better understanding of the learning process and the impact of their teaching upon it." (Angelo, p. xiv, 1993)

Classroom Assessment is an approach designed to help teachers find out what students are learning in the classroom and how well they are learning it. This approach is learner-centered, teacher-directed, mutually beneficial, formative, content specific, ongoing, and firmly rooted in good practice (Angelo, p. 4, 1993).

Assessment that takes place "along the way" in the process of learning and teaching a topic is called "formative". This would be the feedback teachers get when seeing how students are doing as a topic is being presented and learned. "Summative" evaluations are the types of assessments that are given at the end of a topic, chapter, semester. They sum the knowledge of the student up to that time.

The assessments in this book are both formative and summative. Some are intended to give the instructor feedback “along the way”. Some assessments measure when students are “at the gate”.

Responding to Assessment

I think that responding to assessment is the most important idea of this entire book! The purpose of "formative" assessment is to collect useful information about how the students are learning, how the teaching is going, etc., and possibly make adjustments to the course. As we teach we collect informal "formative" assessments in class. We give quizzes, “read” expressions on students’ faces, and get a “feel” for how the class is going. If we sense that “they aren’t getting it”, we might go over a point or a lesson again. Sometimes we give more formal formative assessments, for example, a quiz over some homework problems. If we get concrete evidence that the class overall has high comprehension of the material, we feel confident to continue along the curricular line. But what if 40%, 50%, 60% or more of the class has low or no comprehension? We do not want to halt the progress of the class to re-teach a concept, but we know that there is a problem and we will see that problem manifest on the next exam!

How could we as educators best use the process of assessment? It becomes necessary to view the assessment results as a guide to teaching. In that way we use the assessment results to measure how the class is comprehending a topic. We can then take assessments and quantify them to observe the frequencies of student outcomes.

For example, let’s say you gave a 5-point assessment on content. After scoring the assessments, find the frequencies of those that understood and those that did not understand the content based on the results of the assessment. If the vast majority of the class did well with the material, it makes sense to continue with the course work. But what if 30%, 40%, or more did not master the concept? Can we afford to alter the curriculum to let the other students catch up? Possibly a shift in pedagogy can allow us to answer with a yes, and not compromise the curriculum.

The following possibilities exist:

1. Take 5 - 10 minutes in class for students to pair-share.

Have students work in groups of two: one that passed and one that did not pass the quiz. Let them discuss the problems. Then quiz them again but do not grade the quiz. Students can see their own progress.

2. Give one of the informal formative assessments such as The Muddiest Point, One Minute Feedback or Directed Paraphrasing. Use the assessments to narrow the area of confusion. Then take a few minutes in class to clarify the concept.

3. Make study groups responsible to discuss results of assessments and make sure everyone understands their mistakes. Follow up by having the study group give themselves a participation point if everyone understands.

4. For exams, give a "group test". After all students have taken the exam and have handed in their work, have students retake the same exam at home or in study groups outside of class. Have them compare answers. Collect these "group tests" and give students an extra few points if THE ENTIRE EXAM IS CORRECT AND COMPLETE. I hand back their corrected exam when they hand in their group exam. Students then have very few questions on the exam. They have answered them all for themselves.

5. TELL THE CLASS what you found whenever you give an assessment. If overall the class has not mastered the objective, involve them in some decision making regarding the course and class time. Remind them that the curriculum for the course will still need to be covered. Follow up on the decision you and the class made. I like to remind the students that, like the U.S. president, the teacher has final veto on any suggestions. This is the riskiest of all the above possibilities because you do not know what the class will want. I would only do this if I had developed a good rapport with the class and felt like we all had the same objectives in mind: to learn the material as effectively as possible and to not compromise the course curriculum.

Think of the process of quantifying student outcomes as the same as students asking for help on a homework problem in class. We explain it again, even though they were suppose to understand it for their homework. We explain it because someone asked. Giving formative assessments most, if not every class session, will allow us to have a more holistic picture of the class learning.

The Assessment Templates

The assessments presented in this booklet are largely the author's. However, like all good teachers, many ideas have been "borrowed" at workshops, from other educators and from years of teaching using a

variety of texts and materials.

The assessments are categorized in three ways:

1. The **Standards** from "*The Crossroads in Mathematics: Standards for Introductory College Mathematics Before Calculus*". Note: each Standard has many components to it.
2. The cognitive level of the assessment based on **Bloom's Taxonomy of Learning.**
3. The "True Color" of the assessment. "**True Colors**" is a trademark. Don Lowry developed this system of classifying personality types. His work is based on the work of Jung, Myers-Briggs, and others.

Also included are methods for grading these assessments that follow. I do not believe that if the educators spend more time grading, the students will become better thinkers. Consequently, in each assessment, I have included ways to minimize the time spent on grading assessments. I hope you will find these suggestions helpful.

Following many assessments is a grading rubric for that assessment. It is included on a page standing alone so the user may easily copy it for classroom use. Each rubric may also be found in Appendix B: Grading rubrics. These rubrics are intended to be used by instructors.

Assessment for Intellectual Development

I – 1a

Standard I – 1: Problem Solving

(a) Recognize inappropriate assumptions

Bloom's Taxonomy (Level) of Learning

Evaluation

True Colors™: this assessment will appeal to the following colors because;

Green is logical

Gold likes the content to be clearly defined

Orange likes to put to immediate use what is learned

True Colors: this assessment might be difficult for the following colors because;

Blue turns off if (potential) conflict arises

Materials: Use a medium level word problem for this assessment.

Time it will take to administer: Approximately 15 minutes.

In-class or out-of-class assessment, either, or both: In class

Procedure: Write out a solution to a medium level word problem yourself. Make sure you make at least one inappropriate assumption in the process of reaching the solution. Copy your work for students to investigate. Have students find the inappropriate assumption(s).

OR

Save a student's work on a word problem that was handed in. This work should show some assumptions that were made inappropriately. Copy the problem and the student's work, making sure to never expose the author's name. Have students find the inappropriate assumption(s).

Have students find the flaw(s) in either of the above problems in small groups or individually. I like to copy one problem on half a piece of 8.5 X 11 paper. This cuts down on paper use. Leave space next to the problem for students to make comments. Hand out one copy to each student and one extra copy for the group. Have students work in small groups and make corrections to their paper. When they are finished, they may make a master on the extra copy and hand in that work.

Directions to the students:

Pretend you are the professor for this course and you are grading homework problems. Not only do you want to show the students their errors, but you want to help them understand any mistakes

they may have made. For each correction, write in words what was incorrect about the work. Also write in words what **you** did that was correct.

- Grade the problem in front of you
- Find any incorrect assumptions in the work shown
- Make corrections on the work
- Explain all corrections in writing

Ask the group to hand in the extra copy with corrections they have all agreed upon. Everyone's name in the group should be written on the corrected work. (Remind them to write their first and LAST name.)

Data tabulation: Each correction is worth one point. Not only must the correction be made, but it also must be explained. Both parts must be present to earn a point.

Look at the frequencies of success for this activity. Overall, was the class able to recognize inappropriate assumptions? If not, make a determination for your response to this assessment. See the section on ***Responding to Assessment***.

Assessment for Intellectual Development
I – 1b

Standard I – 1: Problem Solving
(b) Intellectual Risk Taking

Bloom's Taxonomy (Level) of Learning
Application

True Colors™: This assessment will appeal to the following colors because;
Green likes the intellectual challenge
Orange likes the creativity and unusual approaches
Blue is imaginative

True Colors: This assessment might be difficult for the following colors because;
Gold likes the content and structure to be clearly defined

Author's comments: As I thought about this standard, I realized that I was confusing *intellectual risk taking* and *emotional risk taking*. I see intellectual risk taking as a willingness to try different and unknown approaches to solving a mathematics problem. *Emotional risk taking* in a mathematics class may be the willingness to try something, even though it may be wrong, or to answer a question aloud in class even if the answer is incorrect.
Just a note: When students answer questions incorrectly orally in class, I sometimes respond with "Now that is the correct answer to a different question". That usually gets a laugh and also can ease any embarrassment for the student.

Materials: Select a problem that requires multiple steps to solve. It could be an application problem or a computational problem.

In-class or out-of-class assessment, either, or both: Either

Time it will take to administer: Approximately 15 to 20 minutes in class.

Procedure: For out-of-class individual assessment Select a problem that requires multiple steps to solve. It could be an application problem or a computational problem. Post it either on your web-site or in a place students can come to see the problem and solve it. A convenient place might be in the tutoring and testing center on campus.

In the case of posting it in a physical location, students would solve the problem individually. They then leave their work for the subsequent students to see. Each student that comes in will look at all previous solutions posted by other students. S/he then solves the problem using a method not previously done. The work must show a significantly different approach. Significantly different means:

- Not more than 3 steps being the same as anther student' or
- Half of the explanation must be different from other students' work

You may wish to allow for the 4 modalities of solving problems: graphically, numerically, symbolically or algebraically, and verbally or as a written explanation. However, the student's work may also show a significantly different process from another student's.

Ask the staff wherever the problem is posted to write down the name, date and time a student comes in to solve the problem.

Data tabulation: Since this process will be difficult to monitor, I would assign this as an extra credit problem. The students that get to the problem later in the sequence will have more of a challenge than the students that got to the problem early. You could have the students solve this on a first-come-first-serve basis. In that case you will want to give a cut off date for the problem, or a cut off number. You could then grade the problems assigning points for a different approach and for the correctness of the process. I would make this worth a small number of extra credit points.

Another approach could be a sign-up sheet for the day and time to solve the problem, again posting it in a study or testing center. The later in the sequence they sign up, the more potential points they can earn. This would allow the more creative students to really challenge themselves. Here again, the total points earned would be a small number. I find that even 5 points will motivate students. You may wish to allow for only a certain number of slots. This will also limit the number of problems to grade.

In the case of posting the problem on a web site, you may want to say that the first 10 responses will be graded, or whatever your tolerances is for grading.

In-class group assessment

Procedure: For in-class group assessment: Select a medium level problem for this assessment. It could be an application problem or a computational problem. Display the problem. Ask the groups to solve the problem two or three different ways, depending on the amount of time you wish to spend in class. You may wish to allow for the 4 modalities of solving problems: graphically, numerically,
symbolically or algebraically, and verbally, which I would accept as a written explanation.

Grading: I ask for one paper from each group with each person's name in the group listed. Remind students you need first and last names. Grading one paper for every three or four students cuts down on grading time. Everyone in the group earns the same grade. Since it is not possible to verify that each individual can reproduce this work, I assign only a small number of points for the work.

Data tabulation: It may be fun to create a frequency diagram to show how often students used a particular method for solving the problem. Either on the board or on a paper flip

chart draw the dependent and independent axes and label them. List the **methods** used to solve the problem along the independent axis and the **frequencies** along the vertical access. A quick in-class way to do this is as follows:

- require students to have 2" x 2" sticky pads. (Any size will do as long as everyone has the same size)
- students write the name of each method used to solve the problem on one sticky paper, i.e. one paper for each method.
- Have individuals post their papers in a vertical column to show the frequencies of each method used.

You then can see if students seem to be favoring one or two methods. You may find you wish to emphasize alternative problem solving modalities.

Assessment for Intellectual Development
I – 1c

Standard I – 1: Problem Solving

(c) Posing Questions

Bloom's Taxonomy (Level) of Learning

Analysis

True Colors™: this assessment will appeal to the following colors because;

Blue likes language

Green is conceptual

True Colors: this assessment might be difficult for the following colors because;

Gold likes routine

Orange likes looking at the whole picture

Author's Comments: The ability to ask appropriate questions about a topic should be a good indicator of a student's understanding of a problem, concept or process.

Materials: Copy the grading rubric below and hand it out with the assignment. Tell students to hand the rubric back in with the assignment. In this way students will see before hand how they will be evaluated, and you will only need one copy of the rubric for each student.

In-class or out-of-class assessment, either, or both: Out-of-class.

Time it will take to administer: 5 minutes to explain in class.

Procedure: Determine an area that you think students are having difficulty. This may be:

1. a *concept*, i.e. why the graph of a quadratic function with no real solutions would not intersect the x-axis
2. a *process*, i.e. the procedure for graphing a linear equation from a given point and a slope
3. a *problem;* the problem may be a computational or an application.

In any of the above cases, the problem may involve the graphing calculator.

Let's take the first scenario; a concept, i.e. The graph of a quadratic function with no real solutions would not intersect the x-axis. You, the instructor would describe the concepts. In this case, you might say that you are thinking of a quadratic function that has no Real solutions. Or you may draw an appropriate graph of this situation. Ask students to write down five questions that begin with the following words: Why, What, When, Where and How.

The second scenario is a process, i.e. the procedure for graphing a linear equation from a given point and a slope. Tell students that you want them to pose questions that begin with the following words: Why, What, When, Where and How regarding the procedure for graphing a linear equation from a given point and a slope.

The last scenario is a problem. The problem may be a computational, an application, or a problem involving the graphing calculator. For example let's say the problem is: given a linear and a quadratic equation find any solution(s) for the system. Rather than have students solve the problem, ask them to write down five questions that begin with the following words: Why, What When, Where and How.

Scoring and Data tabulation: Make the grading easy on yourself. Let the assignment be worth 5 points, or 3 points, depending on your grading scheme. Use the following rubric for grading:

GRADING RUBRIC FOR THE ASSIGNMENT OF "POSING QUESTIONS"

To earn Full points, all of the following criterion must be met:
__All questions are worded clearly
__The level of difficulty of the questions were medium to difficult
__The author demonstrated understanding of the problem by posing appropriate questions
__Questions show some uniqueness, creativity or deep understanding of the problem through the questions
__Paper was neat, typed, neatly organized, included the description of the assignment, the problem and the questions

• To earn half the possible points, the paper must have at least 3 of the above criterion met.
• Papers that meet only one or two of the above criterion will earn no credit for the assignment.

At this time you may collect very interesting information about the students understanding of their own questions. Tell students that you will select some of the questions from this exercise for their next quiz, test etc. I like to tell the study groups to get together and answer each other's questions and suggest they limit their time for this task to 15 to 20 minutes. Give them out of class time to get together to review questions also. Then actually use a few of the questions on a quiz. To make quiz grading easy, see the appendix on 3-part paper assessments.

GRADING RUBRIC FOR THE ASSIGNMENT OF "POSING QUESTIONS"

To earn Full points, all of the following criterion must be met:
__All questions are worded clearly
__The level of difficulty of the questions were medium to difficult
__The author demonstrated understanding of the problem by posing appropriate questions
__Questions show some uniqueness, creativity or deep understanding of the problem through the questions
__Paper was neat, typed, neatly organized, included the description of the assignment, the problem and the questions

• To earn half the possible points, the paper must have at least 3 of the above criterion met.
• Papers that meet only one or two of the above criterion will earn no credit for the assignment.

Assessment for Intellectual Development
I – 1d

Standard I – 1: Problem Solving
(d) Organizing information

Bloom's Taxonomy (Level) of Learning
Analysis

True Colors™: this assessment will appeal to the following colors because;
Blue is imaginative
Green is curious
Orange likes being active

True Colors: this assessment might be difficult for the following colors because;
Gold likes routine and structure

Author's comments: I usually have a group *assignment* each semester. This *assignment* is an in-depth work that take a few weeks to complete. It can be a complex problem, a research or position paper, or an application problem. The application can be a "Real World" problem. Assessments I-1d, I-1e, I-1j, I-1k are appropriate for a complex problem, an application problem or a research topic. All 4 assessments can be applied to this assignment.

The problem used for these assessments should be open-ended and require multi-steps. Be sure there is enough complexity that will require some data organization. You can use a project type problem at the end of a chapter. You may wish to find an actual problem to be solved, satisfying the **Standard P-5: Experiencing Mathematics**. "Mathematics faculty will provide learning activities, including projects and apprenticeships, that promote independent thinking and require sustained effort and time so that students will have the confidence to assess and use needed mathematics and other technical information independently, to form conjectures from an array of specific examples, and to draw conclusions from general principles." (Crossroads, p. 17, 1995)

In class or Out of class assessment: Out of class group problem

Time to administer: In class discussion of the directions will take 3 – 5 minutes.
Outside of class will take 1 – 2 weeks for assessment I – 1d.

Materials: Use the same problem for assessment I – 1d, I – 1e, I –1j and I-1k.

Procedure: Select an open-ended problem for this assessment. It should also be a multi-step problem. Be sure there is enough complexity that will require some data organization.

Tell students that the first part of the assignment is to organize the data from the problem. At this point they are not required to solve the problem. Show them the grading rubric below. Let them know ahead of time that this is how they will be graded. Give the group sufficient time to write

out and organize the data from the problem. You may wish to offer the option for students to present their data to the class. They should have a way to visually share their organization scheme with the class. A storyboard works well here.

OR

If there is a section on statistics in your course use a more complex assignment. You may assign research topics or have study groups select topics. Have students search the web for information on the selected topic. Their assignment is to organize and present the data in an understandable way. Use the following rubric for grading. Let students see the rubric before grading. Topics can range from cost for vacation rentals, to makes and models of trucks, to grade point averages of students in the certain disciplines. Anything that is appropriate for the course and has interest for the students.

Scoring: Put a check next to each criterion that is met

Rubric for scoring data organization

Exemplary: Full points, to above two thirds of the available points

__Presentation is clear, attractive and understandable
__Data sets are clearly defined and adequately explained
__The relationships that tie the data together is clearly explained

Adequate: Middle number of points, or the middle third of the available points

__Presentation is understandable
__Data sets are clearly defined but not adequately explained
__The relationships that tie the data together is not clearly explained

Inadequate: One third of the available points

__Presentation is not understandable
__Data sets are defined but not explained
__The relationships that tie the data together is not clearly explained

Data tabulation: Create a frequency diagram with the score of exemplary, adequate and inadequate as the independent variable. Show the results to the class. Does the class need more work on drawing diagrams?
A quick in-class way to create a frequency diagram is as follows:
• require students to have 2" x 2" sticky pads. (Any size will do as long as everyone has the same size)
• students write their grade on one sticky paper, i.e. one paper for each person.
• Have individuals post their papers in a vertical column to show the frequencies of each grade.

Rubric for scoring data organization

Exemplary: Full points, to above two thirds of the available points

__Presentation is clear, attractive and understandable
__Data sets are clearly defined and adequately explained
__The relationships that tie the data together is clearly explained

Adequate: Middle number of points, or the middle third of the available points

__Presentation is understandable
__Data sets are clearly defined but not adequately explained
__The relationships that tie the data together is not clearly explained

Inadequate: The bottom third of the available points

__Presentation is not understandable
__Data sets are defined but not explained
__The relationships that tie the data together is not clearly explained

above criterion has not been met.

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Webb, Campbell, Schwartz and Sechrest, *Unobtrusive Measures* (1996) suggests that assessment of performance and quality can often be approaches along novel and unobtrusive paths.