Equilibrium of Concurrent Forces (Force Table)

Objectives:

- Experimental objective
  - Students will verify the conditions required (zero net force) for a system to be in equilibrium under the influence of coplanar forces, and confirm Newton's first law of motion.
- Learning objectives (students should learn...)
  - The concept of vectors and scalars.
  - Graphical and analytic methods for vector addition.

Equipment list:

Force table (table, ring, string, and four 50g hangers), bull's-eye level, set of slotted weights, protractor, and ruler.

Apparatus:

Theory: Vectors vs. scalars – a scalar is a value which has only a magnitude and units (when applicable), like 200, 3m, $25, etc. In many cases, though, physical properties require a value with more information to convey the information. One type of such a value, used extensively in physics, is called a vector. A vector is conveyed as a magnitude and a direction. Many physical properties that are described as vectors also have a scalar counterpart, like velocity (vector) and speed (scalar). The speed of an object can be described for example as 5m/s, but often that is not enough information – the object can be moving in any direction, up, left, south, etc. The velocity of the same object, however, includes that information – i.e. 5m/s at 40° east of north.

There are three standard notations that denote a vector. Typically in handwriting, an arrow is placed over a variable, like “\( \vec{A} \)”, whereas in print, it is usually bold faced, like “\( \mathbf{A} \)”. The third notation can be used in either case, but is becoming much less frequently used, and is called “black-board bold,” and looks like
“A”. The standard boldface will be used to denote a vector in the text for this class. You will need to adopt and consistently use one of the methods above whenever writing a vector. Graphically, a vector is drawn as an arrow, with the direction the arrow points representative of the vector direction, and the length representative of the vector magnitude.

Vector addition – as long as the magnitude and direction is unchanged, a vector can be slid along its line of action or translated (moved in the x, y, and/or z directions) without changing the vector. This is the basis of the geometrical approach to vector addition. There are two slightly different, but equivalent geometrical methods to add two vectors. Take \( \mathbf{A} + \mathbf{B} = \mathbf{R} \), where \( \mathbf{R} \) is called the resultant vector of the vector sum of \( \mathbf{A} \) and \( \mathbf{B} \).

With the triangle method, the tail of \( \mathbf{B} \) is placed at the head of \( \mathbf{A} \). These vectors must be drawn to scale, with the magnitude and direction measured carefully. \( \mathbf{R} \) then, is the vector (with proper magnitude and direction) that starts at the tail of \( \mathbf{A} \) and ends at the head of \( \mathbf{B} \) (see figure 1).

The parallelogram is very similar. As before, all vectors must be drawn to scale. This time, though, \( \mathbf{A} \) and \( \mathbf{B} \) are placed with their tails in the same place. Then a line is drawn from the head of \( \mathbf{A} \), parallel to \( \mathbf{B} \), and another line is drawn from the head of \( \mathbf{B} \), parallel to \( \mathbf{A} \), the lines will intersect, completing a parallelogram. \( \mathbf{R} \), in this case, is the vector (with proper length and direction) that begins at the tails of \( \mathbf{A} \) and \( \mathbf{B} \) and ends at the intersection of the added lines (see figure 2).

The triangle method can be used to show that vector addition obeys the commutative property, that is \( \mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A} = \mathbf{R} \). In figure 1, \( \mathbf{A} + \mathbf{B} = \mathbf{R} \) is shown. Figure 3 now shows \( \mathbf{B} + \mathbf{A} = \mathbf{R} \); notice that the vectors (especially \( \mathbf{R} \)) are unchanged. This shows that vector addition commutes. The triangle method can also be extended to include more vectors, creating a vector polygon, which can also be used to clearly demonstrate this property. To create a vector polygon, for example, \( \mathbf{A} + \mathbf{B} + \mathbf{C} + \mathbf{D} = \mathbf{R} \), simply place each vector with its tail at the head of the previous vector, and the sum is the vector that starts at the tail of the first vector and ends at the head of the last vector (see figure 4).

Another important property of vector addition that can be demonstrated through any of these graphical methods is the associative property, that is \( (\mathbf{A} + \mathbf{B}) + \mathbf{C} = \mathbf{A} + (\mathbf{B} + \mathbf{C}) \). Demonstrating this property is left as an exercise for the student.

Vector addition is, in essence, the same as replacing two or more vectors with a single vector that has the same effect as the multiple vectors combined. Often, however, it is useful to do the reverse; replace a single vector with a sum of multiple vectors. Specifically, replacing it with two or three orthogonal vectors (usually along the Cartesian x, y, and z axes) is called resolving the vector into its component form and is very useful for analytical vector operations such as adding and multiplying. In an x-y plane, the magnitude of a vector is given as \( A_x = |\mathbf{A}| \cos \theta \) (1) and \( A_y = |\mathbf{A}| \sin \theta \) (2), where \( \theta \) is the angle of the vector counterclockwise from the x-axis (for this reason, it is usually best to define 0° and the positive x-axis as the same direction). Note: the magnitude of a vector can be written “shorthand” as normal text (i.e. without the arrow, bold face, or blackboard bold face); for the sake of clarity, in this text, the
magnitude of a vector will be written explicitly in the form \( |\mathbf{A}| \). Since \( A_x \) and \( A_y \) are the magnitudes of the \( x \) and \( y \) components, respectively, they are scalars. Thus a vector written as components takes the form \( \mathbf{A} = A_x \mathbf{\hat{x}} + A_y \mathbf{\hat{y}} \) (3), where \( \mathbf{\hat{x}} \) and \( \mathbf{\hat{y}} \) are unit vectors (magnitude 1 and direction along their respective axes).

Figure 5 is the graphical representation of equation 3; notice that it is the same as the parallelogram in figure 2 in the special case with right angles.

Once a vector has been resolved into its components, analytical vector addition becomes possible. To add (or subtract) vectors analytically is similar to “grouping like terms” in an algebraic expression; add/subtract the magnitudes in the \( \mathbf{\hat{x}} \) direction and do the same for the other directions. For example, if we have two vectors in component form, \( \mathbf{A} = A_x \mathbf{\hat{x}} + A_y \mathbf{\hat{y}} \) and \( \mathbf{B} = B_x \mathbf{\hat{x}} + B_y \mathbf{\hat{y}} \), then the sum is \( \mathbf{A} + \mathbf{B} = (A_x + B_x) \mathbf{\hat{x}} + (A_y + B_y) \mathbf{\hat{y}} \), where the resultant, \( \mathbf{R} = R_x \mathbf{\hat{x}} + R_y \mathbf{\hat{y}} \).

Because vector operations in component form become algebraic expressions, they also obey the rules and properties of algebra. In this form, the associative and commutative properties of vector addition can be easily shown.

While the component form of a vector makes it easier to operate on, the down side is that it is more difficult to interpret the physical significance of the vector than if it were in the form of a magnitude and angle. To change a component form vector into magnitude/angle form takes two steps. First, the magnitude is found by adding the component magnitudes in quadrature (this is the same thing as Pythagorean’s theorem or the distance formula)

\[
|\mathbf{R}| = \sqrt{R_x^2 + R_y^2}
\]

Then, the angle is found by one of the three following inverse trigonometric functions:

\[
\theta = \cos^{-1} \left( \frac{R_x}{|\mathbf{R}|} \right) \quad \text{or} \quad \theta = \sin^{-1} \left( \frac{R_y}{|\mathbf{R}|} \right) \quad \text{or} \quad \theta = \tan^{-1} \left( \frac{R_y}{R_x} \right)
\]

Typically, the inverse tangent is the best equation to use because it does not depend on the magnitude \( |\mathbf{R}| \), but each of these functions are undefined at certain points, so the function you use will depend on the specific circumstances. Also, because of the cyclic property of sine and cosine functions, the value returned could be an unreasonable one. If one function is undefined or returns an unreasonable value, one of the other functions will return the proper value. It is important to consider the value that any function returns, and whether it makes any sense, physically.

Equilibrium – Newton’s first law of motion states that an object in uniform motion will stay in uniform motion, and an object at rest will remain at rest unless it is acted upon by an external net force. We will focus on the object at rest. Mathematically, Newton’s first law is expressed as \( \mathbf{a} = 0 \) iff \( \Sigma \mathbf{F} = 0 \). Iff means “if and only if;” an iff statement is always true backward or forward, whereas an if statement is true forward, but not necessarily true backward. For example, the statement “a shape is a rectangle if it is a square” is true, but backward, “a shape is a square if it is a rectangle” is not necessarily true. The use of “iff,” though makes the statement true either way. Thus, \( \Sigma \mathbf{F} = 0 \) iff \( \mathbf{a} = 0 \) is still true.

Newton’s first law is complimented by his second law of motion (this law will be examined in detail in a later experiment), which is often simplified to the expression \( \mathbf{F} = m \mathbf{a} \), or net force is equal to mass times acceleration. Notice that force and acceleration are both vectors, but mass is a scalar; it simply scales the magnitude of acceleration... hence the term scalar. Force and acceleration always point in the same
direction. Net force means the sum of all forces acting on the object. Using these two laws, we can say that if the net force on an object is equal to zero, the object will not experience an acceleration. This is the condition required for equilibrium of concurrent forces. So, if an object is set at rest in an equilibrium condition, it will not accelerate, and thus it will remain at rest. And this is how you will test for equilibrium.

The term equilibrium means that the parts of a system are in balance. This can be used to describe many different types of systems. For example, in chemistry, chemical equilibrium is used to describe a situation where a reversible reaction does not occur because the concentrations of the chemicals on both sides of the equation are balanced. In this experiment, we are concerned with balancing coplanar forces. If the forces are balanced, the sum of the vectors or the net force is zero, and equilibrium is reached.

There are different types of equilibria; stable, unstable, and metastable. Imagine a marble in a bowl with a round bottom. The marble can sit happily at equilibrium in the bottom of the bowl. Now, if it is pushed to one side or set part way up one of the sides of the bowl, a restoring force will make it roll back and forth until it eventually settles back to its equilibrium in the bottom of the bowl. This is a stable equilibrium. Suppose, though, that the bowl is upside down and the marble is set on top. It is possible to balance the marble so that it is in equilibrium and it will not move. But if the marble is bumped ever so slightly, it will roll away, and not return to equilibrium on top of the bowl. This is unstable equilibrium. Metastable equilibrium is half and half. One metastable configuration is like in figure 6c. It is stable to the left and unstable to the right. The equilibrium in this experiment will be stable (think about why this would be true), which will be used to confirm equilibrium has been reached.

**Procedure:**

You will be hanging masses off of the force table via the pulleys to create forces on the center ring. The edge of the table is marked with angle graduations to measure the direction of the force vector. Because mass, \( m \), is a scalar, it needs to be multiplied by the acceleration of gravity, \( g = 9.81 \text{m/s}^2 \), to find the magnitude of the force it is exerting on the ring, \( |\mathbf{F}| = mg \) (notice this comes from Newton's second law of motion). Be sure to use SI units in your calculations; mass has the units Kg, which multiplied by \( \text{m/s}^2 \) from acceleration gives Kg*\( \text{m/s}^2 \)=N (Newtons), the unit for force.

By balancing three and four forces around the table, you will find a configuration for equilibrium of the forces. To verify equilibrium has been achieved, ensure the strings come off the ring radially and run true over the pulleys, and then pull the ring to the side so it is touching the center pin. When you let go of the ring, it will readjust itself to be centered. Ideally, the ring should be centered on the table to be in equilibrium, but for the purpose of this experiment, it is sufficient as long as the ring is not touching or resting against the pin (the closer to center you can get the ring, the better your results will be).

Because of the “fudge factor” in the position of the ring at equilibrium, it is necessary to find a value called “departure from equilibrium” (DFE). This is the amount of force required to move the ring from perfectly centered to touch the pin (no longer considered in equilibrium), and it tells you how sensitive your apparatus is, similar to uncertainty. Ideally, at equilibrium, the net force should be zero N, but you will probably find that it is not quite zero N. A net force magnitude smaller than the DFE is considered to be within the accuracy of the apparatus.
Part 1 - DFE
1. Place a hanger on the end of each of three strings. Leave the fourth string coiled on top of the table.
2. Set each pulley at 0°, 120° and 240°, respectively, and add 50g to each hanger.
3. Verify that the strings come off the ring radially and run true over the pulleys.
4. At this point, the ring should be perfectly centered on the table. Add small masses to one of the hangers to determine the smallest amount of mass that must be added to make the ring touch the center pin.
5. Multiply the small added mass by the g to find the magnitude of the force required to take the system out of equilibrium. This magnitude is your DFE.

Part 2 – Three forces
6. Remove all added mass from the hangers.
7. Move the pulleys to “crazy” angles (i.e. avoid multiples of 30° and 45°… the crazier the angle, the more interesting your results). Note: you may leave one pulley at 0°, which will save you some work later on.
8. Add mass to the hangers to move the ring so that it is as close to center as you can get it.
9. Verify that equilibrium has been reached by pulling the ring to the pin and releasing it as described at the beginning of the procedure section.
10. Multiply each total mass (hanger mass + added mass) by g to find the magnitude of force.
11. Label your force vectors \( A, B, \) and \( C \) (it doesn’t matter which one is which) and record the directions and magnitudes in table 1.

Part 3 – Four forces
12. Add the fourth pulley to the table and attach a hanger to the fourth string.
13. Repeat steps 6-11, recording the four vectors (labeled \( A, B, C, \) and \( D \)) in table 2.

Data analysis
14. Use the triangle or parallelogram method to find \( \mathbf{R}_1 \) by adding vectors \( A \) and \( B \) from part 2. This diagram must be to scale (be sure to denote the scale in the diagram) and be a full page in size.
15. Compare the resultant vector, \( \mathbf{R}_1 \), to vector \( C \), compare the magnitudes and find the difference in the angles (should ideally be 180°). Record this in table 3.
16. Resolve the three vectors in table 1 into their x and y components.
17. Add the components in table 1 to find the net force \( \mathbf{R}_2 \) and convert it into a magnitude and angle.
18. Use a vector polygon to find \( \mathbf{R}_3 \) by adding vectors \( A, B, C, \) and \( D \) from part 3. This diagram must also be to scale (denote the scale in the diagram) and be a full page in size.
19. Record the resultant, \( \mathbf{R}_3 \), in table 3.
20. Resolve the four vectors in table 2 into their x and y components.
21. Add the components in table 2 to find the net force \( \mathbf{R}_4 \) and convert it into a magnitude and angle.

Prelab Question: To be done as part of your pre lab.
1. List two properties of a vector.
2. Velocity and speed are examples of a vector and a scalar. What other physical quantities can you identify as a vector or a scalar? Label which is a vector and which is a scalar.
3. What is the condition for an object to be in equilibrium (force)?
4. What do you suppose would be the limiting factor in the accuracy of your graphical addition resultant vectors?

Report:

Table 1 – part 2

<table>
<thead>
<tr>
<th>Force</th>
<th>Magnitude</th>
<th>Angle</th>
<th>X component</th>
<th>Y component</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rs (Net force)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2 – part 3

<table>
<thead>
<tr>
<th>Force</th>
<th>Magnitude</th>
<th>Angle</th>
<th>X component</th>
<th>Y component</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>B</td>
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<td>C</td>
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</tr>
<tr>
<td>D</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>Rs (Net force)</td>
<td></td>
<td></td>
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</tbody>
</table>

Table 3 – Results

<table>
<thead>
<tr>
<th>Force</th>
<th>Magnitude</th>
<th>Angle</th>
<th>Discrepancy (magnitude)</th>
<th>Angle difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R3</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Questions:

1. Compare the theoretical values for the magnitude and direction, with the actual magnitude and direction, in steps 14 and 15. Do they match? Why?
2. List all the possible sources of error in determining the resultant force (for both methods).
3. In this experiment, you are led to assume the determined DFE in part 1 is the same as the DFE for parts 2 and 3, but this isn’t true. Why? Why is it ok in to make that assumption in this experiment?
4. You measured a value for DFE, what is the significance of this value?
5. Graphical drafting programs, like AutoCAD, will graph and add vectors, showing the resultant and providing its coordinates. Is AutoCAD adding the vectors graphically and measuring the resultant, or is it adding the vectors algebraically and graphing the resultant? Why?