1 A BEAD MOVING ON A HELIX

A bead moves without friction on a piece of wire which is twisted into the shape of a cylindrical helix (see figure 1). The origin is a center of attractive force varying directly with the distance (constant $k$). Find the equation of motion for the bead sliding on the wire. The radius of the coil is $b$ and the $z$ coordinate is proportional to the coordinate $\phi$, say $z = a\phi$.

2. A SYSTEM OF OSCILLATORS

Consider the vertical oscillations of the system of springs and masses (shown in Figure 2) with the spring constants $k_1 = 78, k_2 = 15$ and $k_3 = 6$ dynes/cm.

(a) Set up the force matrix and find the normal modes, normal coordinates and associated angular frequencies. (b) If the 1g block is displaced by 1cm from its equilibrium position with the 3g block held at equilibrium and both blocks released from rest, describe the subsequent motion of both blocks. (The role of gravity in this problem is restricted to define the equilibrium positions of the masses. It does not affect the vertical oscillations.)

3. ANGULAR ACCELERATION ACCOMPANYING CONTRACTION

A particle of mass $M$ is attached to a string (Fig. 3) and constrained to move in a horizontal plane (the plane of the dashed line). (a) The particle rotates with velocity $v_0$ when the length of the string is $r_0$. How much work is done in shortening the string to $r$? (Hint: The force on the particle due to the string is radial, so that the torque is zero as the string is shortened.)

(b) How does this result compare to the case of a particle rotating on a string that is freely winding up on a smooth fixed peg of finite diameter?

(c) The result of the preceding example has a bearing on the flat shape of many galaxies. How could you use the preceding example to build a simple model of the formation of a galaxy from a large spherical cloud of gas endowed with some angular momentum to start with?

(Hint: It may be preferable to use energy considerations in working out this problem.)
4. Larmor’s Theorem

The force exerted upon a charge $e$ moving with velocity $\vec{v}$ in an electromagnetic field $(\vec{E}, \vec{B})$ is given by

$$m \frac{d^2 \vec{r}}{dt^2} = e \left( \vec{E} + \frac{1}{c} \vec{v} \times \vec{B} \right).$$

Here the vector $\vec{r}$ indicates the position of the charged particle in an inertial system. Show that the effect of a weak uniform magnetic field $\vec{B}$ on the motion of a charged particle in a central electric field $\vec{E}(\vec{r}) \vec{r}$ can be removed by transforming to a coordinate system rotating with an angular frequency $\vec{\omega}_L = -\frac{e}{2mc} \vec{B}$. How weak does the weak $\vec{B}$-field have to be? To normalize notation let us define $\vec{r}' = \vec{a}_{\text{inertial}} + \vec{r}_L$, where $\vec{a}_{\text{inertial}}$ is the instantaneous position of the origin of the central electric field in the inertial frame.

5. Euler’s Angles

A rigid body (e.g. an atomic nucleus - approximately) has its symmetry axes askew to axes defined in the laboratory. In order to align its axes parallel to the lab system it has been found in a particular case that one needs just two Euler rotations: (a) Find the matrix that represents the transformation obtained by rotating about the $x$-axis by $45^\circ$ counterclockwise, and then (b) rotating about the $y'$-axis by $30^\circ$ clockwise. (b) What are the components of a unit vector along the original $z$-axis in the new (doubleprime) system?
Problem 1

Problem 2

Problem 3
Electricity and Magnetism

Answer any four problems. Do not turn in solutions for more than four problems. Each problem has the same weight.

1. **Radiation loss:** When an electron is accelerated, it is losing energy by radiating electromagnetic waves. Using the synchrotron radiation power, find a lifetime of the Rutherford’s atomic model and explain why this model is not adequate for the atomic model in our life. Use the initial radius of the hydrogen atom $r_0 = 0.529 \times 10^{-8}$ [cm]. The electron orbital speed is not relativistic, namely, you can use $\gamma \sim 1$. Direction: 1) Find the total energy of an atom (one electron rounding around a proton), 2) Derive an equation of power loss when the electron’s orbital radius changes, 3) Use the synchrotron power obtain the lifetime of the atom. The lifetime is the time the electron’s orbital radius going down to zero.

![Fig. Q1](image1)

2. **Biot-Savart’s law:** Consider a circular current with a radius $R$ flowing on a conducting wire. (a) Obtain the magnetic fields on axis by Biot-Savart’s law. The wire width is negligible. (b) Place two identical circular coils symmetrically along a common axis, and separated by a distance $R$, which is equal to the radius $R$ of the current (Helmholtz coil). Obtain the magnetic fields on the axis. (c) Show the magnetic field on the axis is almost uniform around the center between two coils.

3. **Gauss’s law:** Two concentric conducting spheres of inner and outer radii $a$ and $b$, respectively, carry charges $\pm Q$. The empty space between the spheres is half-filled by a hemi-spherical shell of dielectric (of dielectric constant $\epsilon/\epsilon_0$), as shown in the figure. (a) Find the electric field between the spheres by the Gauss’s law. (b) Calculate the surface-charge distribution on the inner sphere.

![Fig. Q3](image2)
4. **Boundary condition:** A plane electromagnetic wave \((E \text{ and } H)\) is coming through a material \((\epsilon_1 \text{ and } \mu_1)\), and it is going into another material \((\epsilon_2 \text{ and } \mu_2)\). The wave is crossing the interface of these two materials normally. a) Obtain the reflected wave \((E_1 \text{ and } H_1)\) and the transmitting wave \((E_2 \text{ and } H_2)\). b) Calculate the reflection rate \(R\) and the transmission rate \(T\) of the incoming energy flux. c) Calculate the reflection rate and the transmission rate when the light is crossing the interface of air/glass. The refractive index of the glass against the air is \(n = 1.5\).

![Fig. Q4](image)

5. **Lorentz transformation:** A charge \(q\) is at rest in \(K\)-frame (laboratory) in a static magnetic field \(B = (0, 0, B)\) but no electric field. In the \(K'\)-frame, which is moving in \(x\)-direction with velocity \(v\) against \(K\)-frame, a person ‘\(b\)’ see the charge is moving with \(-v\) in \(x\)-direction. How does the fields look in the moving \(K'\)-frame? Is the charge rotating by \(v \times B\) force in \(K'\)-frame?

![Fig. Q5](image)
Supplements of Electricity and Magnetism

Constant parameters

- Electric permittivity of free space $\epsilon_0 = 8.854 \times 10^{-12}$ (mks) or $1/4\pi$ (cgs)
- Magnetic permeability of free space $\mu_0 = 4\pi \times 10^{-7}$ (mks) or $4\pi/c^2$ (cgs)
- Electron charge $e = 1.6 \times 10^{-19}$ [C] or $4.8 \times 10^{-10}$ [esu]
- Electron mass $m = 0.91 \times 10^{-30}$ [kg] or $0.91 \times 10^{-27}$ [g]

Maxwell equations

\[
\begin{align*}
\nabla \cdot \mathbf{D} &= \rho \\
\nabla \cdot \mathbf{D} &= 4\pi \rho \quad \text{(Coulomb’s law)} \\
\n\nabla \times \mathbf{E} + \frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} &= 0 \\
\n\nabla \times \mathbf{E} + \frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} &= 0 \quad \text{(Faraday’s law)} \\
\n\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} &= \mathbf{J} \\
\n\nabla \times \mathbf{H} - \frac{1}{c} \frac{\partial \mathbf{D}}{\partial t} &= \frac{4\pi}{c} \mathbf{J} \quad \text{(Ampere – Maxwell’s law)} \\
\n\n\nabla \cdot \mathbf{B} &= 0 \\
\n\n\nabla \cdot \mathbf{B} &= 0 \quad \text{(Absence of free magnetic poles)}
\end{align*}
\]

Synchrotron radiation

Power of the synchrotron radiation from a charge $q$ with acceleration $\dot{\beta}$ is given by the following equation. Here $\beta = v/c$, $\gamma = 1/\sqrt{1-\beta^2}$.

\[
\begin{align*}
\text{MKS} & \quad P = \frac{1}{6\pi \epsilon_0} \frac{q^2 \beta^2 \gamma^4}{c} \\
\text{cgs} & \quad P = \frac{2}{3} \frac{q^2 \beta^2 \gamma^4}{c}
\end{align*}
\]

Biot-Savart’s law

Magnetic field $\delta \mathbf{B}$ from a small current $\delta \mathbf{I}$ is

\[
\begin{align*}
\text{MKS} & \quad \delta \mathbf{B} = \frac{\mu_0}{4\pi} \frac{\delta \mathbf{I} \times (\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3} \\
\text{cgs} & \quad \delta \mathbf{B} = \frac{1}{c} \frac{\delta \mathbf{I} \times (\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3}
\end{align*}
\]

here $\mathbf{x}'$ is the location of the current and $\mathbf{x}$ is the observation point.
**Taylor expansion**

Generally the Taylor expansion is given by

\[
f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n.
\]

and the following Taylor expansion is useful for the Helmholz coli problem,

\[
\frac{1}{[(z \pm R/2)^2 + R^2]^{3/2}} = \left[ -\frac{8}{5\sqrt{5}R^3} + \frac{48}{25\sqrt{5}R^4}z \pm \frac{256}{125\sqrt{5}R^6}z^3 - \frac{1152}{625\sqrt{5}R^7}z^4 + O(z^5) \right]
\]

**A plane wave and the refractive index**

A plane wave has the following relation between electric field and magnetic field,

\[
\frac{H}{E} = \sqrt{\frac{\varepsilon}{\mu}}
\]

and the refractive index is given as \( n = \sqrt{\varepsilon^* \mu^*} \), here \( \varepsilon^* (\mu^*) \) is the ratio of the electric permitivity (magnetic permeability).

**Lorentz transformation**

\[
x'_\mu = \Lambda_{\mu\nu}x_\nu
\]

4-dimensional vectors and tensors

- Lorentz transformation matrix to the frame \( K' \) moving in \( X \)-direction with velocity \( \beta \) and \( \gamma = 1/\sqrt{1-\beta^2} \):

\[
\Lambda_{\mu\nu} = \begin{pmatrix}
gamma & -i\gamma\beta & 0 & 0 \\
 i\gamma\beta & \gamma & 0 & 0 \\
 0 & 0 & 1 & 0 \\
 0 & 0 & 0 & 1
\end{pmatrix}
\]

- Space and time: \( x_\mu = (ict, x, y, z) \)
- Charge and current: \( j_\mu = (ic\rho, j_x, j_y, j_z) \)
- Potential: \( A_\mu = (ic\phi, A_x, A_y, A_z) \)
- Velocity \( w_\mu = (ic\gamma, \gamma v_x, \gamma v_y, \gamma v_z) \)
- Lorentz transformation formula of \( E \) and \( B \) fields (transferred by \( \Lambda_{\mu\nu} \)).

\[
\begin{align*}
E'_x &= E_x \\
E'_y &= \gamma(E_y - \beta B_z) \\
E'_z &= \gamma(E_z + \beta B_y)
\end{align*}
\]

\[
\begin{align*}
B'_x &= B_x \\
B'_y &= \gamma(B_y + \beta E_z) \\
B'_z &= \gamma(B_z - \beta E_y)
\end{align*}
\]
Complete 4 out of 5 problems. Each problem has the same weight.

1.) a.) Suppose that \( A \) and \( B \) are two commuting observables. If \( |a_1> \) and \( |a_2> \) are two eigenvectors of \( A \) with different eigenvalues, prove that the matrix element \( <a_1|B|a_2> = 0 \).

b.) Now suppose the matrix

\[
\begin{pmatrix}
-a & 0 & 0 \\
0 & 2a & 0 \\
0 & 0 & 2a
\end{pmatrix}
\]

and

\[
\begin{pmatrix}
b & 0 & 0 \\
0 & 0 & b \\
0 & b & 0
\end{pmatrix}
\]

in the basis \( |1>, |2>, |3> \) where \( a \) and \( b \) are both real constants. Show that \( A \) and \( B \) commute. Is it possible to find an orthonormal set of kets which are simultaneous eigenvectors of both \( A \) and \( B \)? If so, construct such a set, or if not, explain why this is not possible.

2.) The Zeeman Hamiltonian describes the interaction of an atom with an external magnetic field \( B_0 \).

Assume this Hamiltonian can be written as \( (\omega_0) Z = \omega_0 (L_z + 2S_z) \) where \( \omega_0 = -\frac{q}{2m_e} B_0 \) is the Larmor frequency of the atom, \( L \) is the orbital angular momentum of the electron, \( S \) is the spin angular momentum of the electron and \( I \) is the nuclear spin. Let \( F \) be the total angular momentum: \( I = L + S + I \).

a.) For a hydrogen atom in its ground state \( |n=0, l=0, m=0> \) in an external field \( B_0 \) (neglecting fine and hyperfine structure), determine the energy shifts of this level resulting from the magnetic field for each of the spin states \( |m_S, m_I> \) of the electron and nuclear spins. What is the degeneracy of each energy level?

b.) Now add a small perturbation \( A \vec{I} \cdot \vec{S} \) where \( A \) is a positive real constant. Use perturbation theory at the lowest non-vanishing order to determine the energy shifts of the levels you found in part a.) You may assume \( Ah^2 \ll \hbar \omega_0 \). What is the degeneracy of each level?

3.) A spinless particle is bound in a two-dimensional harmonic potential \( V(x, y) = \frac{1}{2} m \omega^2 (x^2 + y^2) \). Let the kets \( |n_x, n_y> \) with \( n_x = 0, 1, 2 \) etc. and \( n_y = 0, 1, 2 \) etc. describe the energy eigenstates of the system. Assume the particle is initially in the first excited state (with the excitation along the y-direction) i.e. \( |\psi> = |0, 1> \). At time \( t=0 \), a perturbation of the form \( W(x, t) = Ax^2 e^{-t/\tau} \) is turned on. Using first-order time dependent perturbation theory, calculate the probability that after a long time \( t >> \tau \), the system will occupy any final state.
4). Consider a system of three distinguishable spin-1/2 particles. Assume the initial state of the system is given by $|\psi\rangle = -\frac{1}{\sqrt{3}} [\uparrow \downarrow \uparrow > + \uparrow \downarrow \downarrow > - \downarrow \downarrow \uparrow >]$. Let the total spin be $\vec{S} = \vec{S}_1 + \vec{S}_2 + \vec{S}_3$.

a.) If $S_{3z}$ is measured, what results are possible and with what probability?

b.) If instead $S_z$ is measured, what results are possible and with what probability?

c.) Suppose instead $S_{3x}$ is measured, what results are possible and with what probability?

5.) A spinless particle of mass $m$ and energy $E = \hbar^2 k^2 / 2m$ scatters from a spherical $\delta$-function shell potential $V(r) = \frac{\hbar^2 \gamma}{2m} \delta(r - R)$ where $\gamma$ is a positive real constant. Determine the differential cross section for scattering $\sigma(\theta, \phi)$ using the first order Born Approximation. Is the result isotropic? If not, indicate the explicit dependence on $\theta$ and $\phi$. 
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Schrodinger’s Equation:
\[ i\hbar \frac{\partial \psi}{\partial t} = H\psi \]

Hamiltonian:
\[ H = -\frac{\hbar^2}{2m} \nabla^2 + V \]

Raising and lowering operators:
\[ \hat{a} = \sqrt{\frac{m\omega}{2\hbar}} \left( \hat{x} + \frac{i}{m\omega} \hat{p} \right) \]
\[ \hat{a}^+ = \sqrt{\frac{m\omega}{2\hbar}} \left( \hat{x} - \frac{i}{m\omega} \hat{p} \right) \]
\[ \hat{a} \mid n > = \sqrt{n} \mid n-1 > \]
\[ \hat{a}^+ \mid n > = \sqrt{n+1} \mid n+1 > \]

Angular Momentum:
\[ J_z \mid j, m > = \hbar \sqrt{j(j+1) - m(m \pm 1)} \mid j, m \pm 1 > \]

Scattering:
\[ f^{(1)}(\vec{k}, \vec{k}') = -\frac{1}{4\pi \hbar^2} \frac{2m}{\hbar^2} \int d^3 \vec{x}' \exp[i(\vec{k} - \vec{k}') \cdot \vec{x}'] V(\vec{x}') \]
1. **Partition function. Two-level system.**
Consider a system composed of a very large number $N$ of distinguishable and mutually non-interacting atoms, each of which has two (non-degenerate) energy levels: $\varepsilon_1=0$ and $\varepsilon_2=\varepsilon > 0$.

a) Using the *microcanonical ensemble*, find the partition function and the entropy (as a function of $E$, $N$, and $\varepsilon$), and the average energy $E$ (as a function of $N$, $\varepsilon$, and temperature $T$).

b) Using the *canonical ensemble*, find the partition function, the Helmholtz free energy $F$, and the average energy $E$ (as a function of $N$, $\varepsilon$, and temperature $T$).

c) Compare the results for the average energy $E$ from (a) and (b) and comment on it.

2. **Formal thermodynamic manipulations.**
From the fundamental thermodynamic relation show that

a) \( \left( \frac{\partial \mu}{\partial P} \right)_{S,N} = \left( \frac{\partial V}{\partial N} \right)_{S,P} \)

where $V$, $N$, $P$, $\mu$ and $S$ are volume, number of particles, pressure, chemical potential, and entropy.

b) \( \left( \frac{\partial S}{\partial P} \right)_{T,\mu} = -\left( \frac{\partial V}{\partial T} \right)_{P,\mu} \)

where $T$ is temperature and the rest of the variables is specified in (a).

*Hint: The fundamental thermodynamic relation is $dU = TdS - PdV + \mu dN$, so the independent variables are $V, N, S$. To prove the desired expressions you need to use a Maxwell relation approach and define the thermodynamic potentials that has independent variables of interest in (a) and (b).*

3. **Equilibrium of vapor and solid.**
A solid exists in equilibrium with its vapor, which is treated as a classical ideal gas. Here we choose the energy reference such that a particle in the gas has zero energy at zero velocity, so the solid has a binding energy $\varepsilon$ per atom. (i.e. the total energy of the solid with $N$ atoms is $U_0 = -N\varepsilon$ at $T = 0$).
a) Use the Einstein model to describe the excitations in the solid above the energy $U_0 = -N\varepsilon$ of the ground state. Find the partition function of one oscillator and Helmholtz free energy of the Einstein solid.

*Hint: Einstein assumed that a solid comprising $N$ atoms can be treated as an assembly of $3N$ distinguishable one-dimensional oscillators with the same frequency $\omega$. Also you need to include the binding energy $\varepsilon$ in the Helmholtz free energy of the solid.*

b) Find the Helmholtz free energy of the vapor.

c) What is the condition for the equilibrium between the vapor and the solid?

d) Find the vapor pressure $P(T)$.

4. **Chain.**
Consider a one-dimensional chain consisting of $N$ molecules which exist in two configurations $\alpha$ and $\beta$, with corresponding energies $\varepsilon_\alpha$ and $\varepsilon_\beta$, and lengths $a$ and $b$. The chain is subject to a tensile force $f$ (see Fig. 1).

![Diagram of a one-dimensional chain with forces $f$ and tensile force $f$.](image)

a) Calculate the average length $<L>$ as a function of $f$ and the temperature $\tau$.

*Hint: you may consider using the following expression $<L> = \tau\left(\frac{\partial \ln Z}{\partial f}\right)$, where $Z$ is a partition function.*

b) Assume that $\varepsilon_\alpha > \varepsilon_\beta$ and $a > b$. Estimate the average length $<L>$ in the absence of the tensile force ($f=0$) as a function of temperature. What are the high- and low-temperature limits and what is characteristic temperature at which the changeover between the two limits occurs?
c) Calculate the linear response function
\[ \chi = (\partial <L> / \partial f)_{f=0} . \]
Produce a general argument to show that \( \chi > 0 \).

5. **Quantum gas.**

The number of bosons \( N_{\text{exc}} \) in volume \( V \) and each of mass \( m \) in excited states are given by the following expression (under assumption that it is a sufficiently large number of particles in the ground state)
\[ N_{\text{exc}} = \frac{V}{4\pi^2} \left( \frac{2m}{\hbar^2} \right)^{3/2} \tau^{3/2} \int_0^\infty \frac{x^{1/2} dx}{e^x - 1} , \]

where the integral is evaluated using the Riemann function
\[ \xi(3/2) = \frac{2}{\sqrt{\pi}} \int_0^\infty \frac{x^{1/2} dx}{e^x - 1} = 2.612. \]

a) Find the expression for the Bose-Einstein condensation temperature \( \tau_C \).
b) Calculate the Bose-Einstein condensation temperature for a vapor of \(^{87}\text{Rb} \) atoms (86.9 amu) with a density of \( 2.5 \times 10^{12} \text{ cm}^{-3} \).
c) For experiment in (b), find the temperature at which half of the atoms are in the condensate.
d) What are the values of the chemical potentials for Bose and Fermi gases at \( \tau \to 0 \)? Compare and comment on your answer.
e) Does boson condensation occur in two dimensions? Provide the detailed explanation.
Supplemental expressions that may (or may not) be useful in this exam

Thermodynamic potentials

\[ E \rightarrow F \]
\[ H \rightarrow G \]

Stirling’s approximation for large \( N \)
\[ \ln(N!) = N \ln(N) - N. \]

Multiplicity, entropy, and partition functions

\[ S = k_B \ln \Omega, \quad Z = \sum \exp(-\varepsilon_i / \tau), \quad < X > = \sum_i X_i P_i \]
\[ \beta = 1/(k_B T) = 1/\tau \quad < E > = -\left[\frac{\partial \ln Z}{\partial \beta}\right]_{V,N} \quad F = -k_B T \ln Z \]
\[ Z = \sum \exp[\beta(N_i \mu - \varepsilon_i)] \quad \mu(T,V,N) = \left(\frac{\partial F}{\partial N}\right)_{T,V} \quad \mu(T,P,N) = \left(\frac{\partial G}{\partial N}\right)_{T,P} \]

Ideal gas
\[ Z = \frac{V^N}{N! \lambda^{3N}}; \quad \lambda = \left(\frac{h^2}{2\pi mk_B T}\right)^{1/2} = \left(\frac{\beta h^2}{2\pi m}\right)^{1/2} \]

Quantum gas
\[ < n_j > = \frac{1}{e^{\beta(\varepsilon_j - \mu)} \pm 1} \]
## Fundamental Physical Constants

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed of light</td>
<td>c</td>
<td>$2.99792458 \times 10^8 \text{ m/s}$</td>
</tr>
<tr>
<td>Planck constant</td>
<td>h</td>
<td>$6.6260755 \times 10^{-34} \text{ J/s}$</td>
</tr>
<tr>
<td>Planck constant</td>
<td>h</td>
<td>$4.1356692 \times 10^{-15} \text{ eV/s}$</td>
</tr>
<tr>
<td>Planck hbar</td>
<td>$\hbar$</td>
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<tr>
<td>Planck hbar</td>
<td>$\hbar$</td>
<td>$6.821121 \times 10^{-16} \text{ eV/s}$</td>
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<tr>
<td>Gravitation constant</td>
<td>G</td>
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<tr>
<td>Boltzmann constant</td>
<td>k</td>
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</tr>
<tr>
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<tr>
<td>Molar gas constant</td>
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<tr>
<td>Charge of electron</td>
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</tr>
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<td>Permeability of vacuum</td>
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<td>Permittivity of vacuum</td>
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<tr>
<td>Coulomb constant</td>
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<tr>
<td><strong>Atomic mass unit</strong></td>
<td>u</td>
<td><strong>931.49432 MeV / c^2</strong></td>
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<td>-------------------------</td>
</tr>
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<tr>
<td><strong>Flux quantum</strong></td>
<td>$\Phi_0$</td>
<td>$2.067834 \times 10^{-15} \text{ T m}^2$</td>
</tr>
<tr>
<td><strong>Bohr radius</strong></td>
<td>$a_B$</td>
<td>$0.529177249 \times 10^{-10} \text{ m}$</td>
</tr>
<tr>
<td><strong>Standard atmosphere</strong></td>
<td>atm</td>
<td>$101325 \text{ Pa}$</td>
</tr>
<tr>
<td><strong>Wien displacement constant</strong></td>
<td>b</td>
<td>$2.897756 \times 10^{-3} \text{ m K}$</td>
</tr>
</tbody>
</table>