Effectiveness of Variable Speed Limits

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Variable Speed Limits (VSLs)

- Commend safe driving speeds during less than ideal conditions
- Considerable efforts have been made to evaluate their effects on reducing crash potentials and enhancing safety
VSL to Improve Traffic Flow

- A widespread scheme to improve traffic flow efficiency on freeways
Implementations

I-5, Seattle, WA

I-4, Orlando, FL
Why It May Help

- No consensus
- One of Primary Mechanisms
  - Mainline metering to regulate inflow to prevent or delay traffic flow breakdown and avoid or postpone capacity drop at active bottlenecks (e.g., Papageorgiou et al., 2006; Carlson et al., 2010)
Capacity Drop

Source: Srivastava and Geroliminis (2013)

Cassidy and Bertini (1999): 4% to 10%
Evaluation of VSLs

- The results of VSL implementations are mixed. Some do not show visible impacts on traffic conditions (e.g., Nissan and Koutsopoulos, 2011; Elefteriadou et al., 2012).
  - Design of the VSL system, such as locations of VSL signs and algorithms to adjust speed limits
  - Drivers’ compliance

- Previous studies mainly focus on their localized impacts on traffic flow and microscopic behaviors of drivers
Evaluation of VSLs (Cont’d)

- This study examines VSL from a systems perspective and considers individual travelers’ long-term response to VSL, e.g., departure time choice. In a stylized setting of morning commute, we show that:
  
  - An ideal VSL system, which adjusts speed limits continuously, may reduce travel time by 2.6 to 11.1% (of the original queuing delay)
  
  - A more practical VSL system, which adjusts speed limits in a discrete manner, will unlikely reduce total travel time, individual and social travel cost
Modeling Morning Commute

- Initialized by Vickrey (1969) for a single bottleneck with homogenous users
- Investigate the formation and dissipation of the rush hour queues
- Extended to more general cases. See, e.g., Arnott et al. (1998) and de Palma and Fosgerau (2011), for recent reviews
Vickrey’s Model

- Work starts at 8:00
- Early/Late Arrival Penalty (Schedule Cost)
- $t_f = l/v_f$
- Capacity = $s << N$
- $\gamma \in [2, 4]$ (Tseng et al., 2005)

$N$ persons

$t^* = 8:00$
Individual Travel Cost

\[ c(t) = \alpha \cdot T(t) + \beta \cdot \max\{0, t^* - t - T(t)\} + \gamma \cdot \max\{0, t + T(t) - t^*\} \]

\[ C(t) = \alpha \cdot (\text{travel time}) + \beta \cdot (\text{time early}) + \gamma \cdot (\text{time late}) \]

where:

\[ T(t) = t_f + q(t)/s \]

\[ \gamma > \alpha > \beta \]
User Equilibrium

- Commuters are assumed to select departure times to minimize their individual travel costs.
- A Wardrop equilibrium will be achieved where no commuter can reduce his or her travel cost by unilaterally changing departure time.
- The travel cost is the same for every commuter.
Departure/Arrival Patterns

Equilibrium individual travel cost:

$$c(N) = \alpha t_f + \delta \frac{N}{s}$$

$$\delta = \frac{\beta \gamma}{(\beta + \gamma)}$$
What If Capacity Drops?

It is assumed that if $q \geq q_c, s' = \lambda s$ where $\lambda < 1$, $q_c < \frac{\delta}{\alpha} N$ or $N > \frac{\alpha}{\delta} q_c$.

Diagram:

- Cumulative
- $N$
- $\delta N/\beta + \varepsilon$
- $\delta N/\beta$
- $r_1' = \frac{\alpha}{\alpha - \beta} s'$, $r_2' = \frac{\alpha}{\alpha + \gamma} s'$
- $q_c$
- $s'$
- Time
- $0, t_s', t^*, t_e'$

Graphical representation of cumulative analysis with points and lines indicating different scenarios and parameters.
Impacts of Capacity Drop

- Longer duration of rush hour
  \[ t'_s = t^* - \frac{\delta N}{\beta s'} + \frac{\delta}{\beta} \frac{\alpha}{\beta} \left( \frac{q_c}{s'} - \frac{q_c}{s} \right) - t_f \; ; \; t'_e = t^* + \frac{\delta N}{\gamma s'} - \frac{\alpha}{\beta + \gamma} \left( \frac{q_c}{s'} - \frac{q_c}{s} \right) \]

- Larger individual travel cost
  \[ c(N) = \alpha t_f + \delta \left( \frac{N}{s'} - \frac{\alpha}{\beta} \frac{q_c}{s'} + \frac{\alpha}{\beta} \frac{q_c}{s} \right) \]

- More commuters will arrive early (for the higher capacity)
  \[ \varepsilon = \frac{\delta}{\beta} \frac{\alpha}{\gamma} \left( 1 - \frac{s'}{s} \right) q_c > 0 \]
Can VSL Help?

- In our setting, a traffic-responsive VSL system functions similarly as a pre-determined time-varying system.
- We are interested in how the system can adjust speed limits over time to possibly avoid capacity drop.
Ideal System

- Each individual driver entering the freeway at $t$ is advised a “personalized” speed limit $\bar{v}(t)$, which remains constant for the particular driver along the freeway segment toward the bottleneck.

- The speed limit varies continuously with time, implying that each individual driver may face a different speed limit.

- 100% compliance
Design and Impact of VSL

FIFO principle satisfied
Bounding Efficiency of VSL

- Total Travel Time

\[ TT(\lambda) = t_f \cdot N + \frac{1}{2} \frac{q_c}{s} \cdot \left[ \frac{\delta}{\beta} N + \frac{\alpha}{\beta + \gamma} \left( 1 - \lambda \right) q_c \right] + \frac{1}{2} \left( \frac{1}{\lambda} \frac{\delta}{\alpha} \frac{N}{s} - \frac{1}{\lambda} \frac{\delta}{\beta} \frac{q_c}{s} + \frac{\delta}{\beta} \frac{q_c}{s} \right) \cdot \left( N_c - \frac{\alpha}{\beta} q_c \right) \]

- Travel Time Saving

\[ \Delta TT(\lambda) = TT(\lambda) - TT(1) = \frac{1}{2} \frac{1}{s} \left[ \left( 1 - \lambda \right) \frac{\alpha}{\beta + \gamma} \left( q_c \right)^2 + \left( \frac{1}{\lambda} - 1 \right) \frac{\delta}{\alpha} \left( N - \frac{\alpha}{\beta} q_c \right)^2 \right] \]

Note: \[ \frac{\partial \Delta TT(\lambda)}{\partial q_c} < 0 \]

\[ 0 < q_c < \frac{\delta}{\alpha} N \]
Bounding Efficiency (Cont’d)

- The percent of travel time reduction

\[ \frac{\delta}{\gamma} (1 - \lambda) \left( \frac{\delta}{\beta} + \frac{\delta}{\gamma} \frac{1}{\lambda} \right) < \frac{\Delta TT(\lambda)}{TT(1) - t_f \cdot N} < \frac{1}{\lambda} - 1 \]

Assume that \( \lambda = 0.9 \) (Cassidy and Bertini, 1999); \( \frac{\gamma}{\beta} = 3 \) \( \left( \frac{\gamma}{\beta} \in [2, 4] \right) \) by Tseng et al., 2005

\[ 2.6\% < \frac{\Delta TT(\lambda)}{TT(1) - t_f \cdot N} < 11.1\% \]

*Similar bounds can be obtained for other performance measures*
First-Best Congestion Pricing
Costs under Congestion Pricing

Cost

Arrival Time

Toll

Arrival Time
Practical VSL

(a) Speed Limit

\[ v_f \]

\[ \bar{v}_{\text{min}} \]

0

Departure Time

(b) Speed Limit

\[ v_f \]

\[ \bar{v}_{\text{min}} \]

0

Departure Time
Proposed VSL Scheme

\[ N_v + \frac{\delta}{\beta} N_c \]

\[ N_v \]

\[ N \]

\[ N_c : \# \text{ of commuters who face constant speed limit} \]

\[ N_v : \# \text{ of commuters who face variable speed limit} \]

\[ \bar{v}_i = \begin{cases} v_f \cdot \frac{t_f}{t_f + (i-1)\Delta t_k} & i \leq k \\ v_f \cdot \frac{t_f}{t_f + k\Delta t_k} & i \geq k+1 \end{cases} \]

\[ \Delta t_k = \frac{1}{k} \frac{\beta}{\alpha} \frac{N_v}{s} \]
Proposed VSL Scheme (Cont’d)

- When \( N_c \) is sufficiently large, capacity drop occurs

\[ \bar{N}_c = \frac{\alpha}{\delta} q_c \]
Total Travel Time

\[ TT(N_c) = TT_o(N_c) + TT_q(N_c) \]
\[ TT_o(N_c) = (N - N_c) \cdot \left( t_f + \frac{1}{2} \frac{\beta}{\alpha} \frac{N - N_c}{s} \right) + N_c \cdot \left( t_f + \frac{\beta}{\alpha} \frac{N - N_c}{s} \right) \]
\[ TT_q(N_c) = \begin{cases} 
N_c \cdot \left( \frac{1}{2} \frac{\delta}{\alpha} \frac{N_c}{s} \right) & \text{when } N_c \leq \bar{N}_c \\
\frac{1}{2} \frac{q_c}{s} \cdot \left[ \frac{\delta}{\beta} N_c + \frac{\alpha}{\beta + \gamma} (1 - \frac{s'}{s}) q_c \right] + \frac{1}{2} l_q(N_c) \cdot \left[ N_c - \frac{\alpha}{\beta} q_c \right] & \text{when } N_c > \bar{N}_c 
\end{cases} \]

- Proposition 1. The proposed VSL scheme can help reduce total travel time if \( \lambda \leq \gamma / (\beta + 2\gamma) \). It cannot reduce total travel time if \( \lambda \geq \gamma / (\beta + \gamma) \), when \( \frac{q_c}{N} \to 0 \).
Effective Region (Region 3)

The proposed VSL system is unlikely to reduce total travel time even if capacity drop can be completely avoided.

The ratio of late arrival penalty to early arrival penalty $\frac{\gamma}{\beta}$

$\lambda = \frac{\gamma}{\beta + \gamma}$

$\lambda = \frac{\gamma}{\beta + 2\gamma}$
Proposition 2. The proposed VSL scheme can help reduce total travel cost if $\lambda < \frac{\gamma}{(\beta + \gamma)}$. If this condition holds, it is socially preferable to set $N_c = \bar{N}_c$
The ratio of late arrival penalty to early arrival penalty \( \frac{\gamma}{\beta} \)

The percentage of remaining capacity \( \lambda \)

Capacity drop (10%)

Region 1

Region 2

Region 3

Line 1

Line 2

The proposed VSL system is unlikely to reduce total travel cost
Total Emissions Cost

- Emissions are estimated using a modal emissions approach (e.g., Matzoros and Van Vliet, 1992; Frey et al., 2001; Coelho et al., 2005).

- A vehicle’s operation is divided into two modes: cruising on freeway and queuing at the bottleneck, each associated with a constant emissions factor for each primary pollutant.

- We use $e_o$ and $e_q$ to denote the monetary equivalents of emissions per unit time associated with cruising and queuing respectively $\rho = e_q/e_o, \rho > 1$
Total Emissions Cost (Cont’d)

\[ TE(N_c) = TT_o(N_c) \cdot e_o + TT_q(N_c) \cdot e_q \]

\[ \rho_1 = \min\left\{ \left( \theta(N) \right)^{-1}, \rho_o \right\}; \quad \rho_2 = \frac{\beta}{\delta} \]

- Proposition 3. To minimize total emissions cost, under the proposed VSL scheme: i) If \( \rho \leq \rho_1 \), then \( N_c = N \); ii) If \( \rho_1 < \rho < \rho_2 \), then \( 0 < N_c < N \); iii) If \( \rho > \rho_2 \), then \( N_c = 0 \).
Reality Check

- $\frac{\gamma}{\beta} = 3$ and $\frac{\gamma}{\alpha} = 1.5$ (Tseng et al., 2005) and $\lambda = 0.9$.
  Considering $\frac{q_c}{N} \rightarrow 0$, we have $1.08 \leq \rho_1 \leq 1.35; 1.2 \leq \rho_2 \leq 1.5$.

- Monetary values of vehicular emissions (US$/hour): $e_q = 1.392; e_o = 0.864$.

\[ \rho = 1.61 \text{ and thus } \rho > \rho_2. \]

The scheme can help reduce total emissions cost and minimize it by avoiding capacity drop.
Total Social Cost

- Proposition 4. The proposed VSL scheme can reduce total social cost if

\[ \lambda < \alpha \left( \alpha + e_o - \frac{\delta}{\beta} e_q \right)^{-1} \frac{\delta}{\beta} \]

It cannot reduce total social cost if, when \( \frac{q_c}{N} \to 0 \),

\[ \lambda \geq \frac{\alpha + e_q}{\alpha + e_o} \frac{\delta}{\beta} \]
Reality Check

• Consider $\alpha = 13.7; \beta = 6.4; \gamma = 20.0$ (Tseng et al., 2005) and $e_q = 1.392; e_o = 0.864$. According to Proposition 4, if capacity drops by more than 23.2%, VSL help reduce total social cost. If the capacity drop is less than 21.5%, it does not help.

_The proposed VSL system unlikely reduces total social cost_
Summary

- VSL can prevent or delay the activation of a bottleneck and thus reduce queuing delay. It can also help reduce total emissions cost.
- There are situations where even a properly-designed VSL system may not reduce total travel time, individual travel cost, and total social cost.
- Caution should be exercised to identify conditions where VSL can help improve traffic flow efficiency.
- VSL may still be a good system to implement to improve safety.
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